DOCUMENT RESUME

ED 036 439 SE 007 831

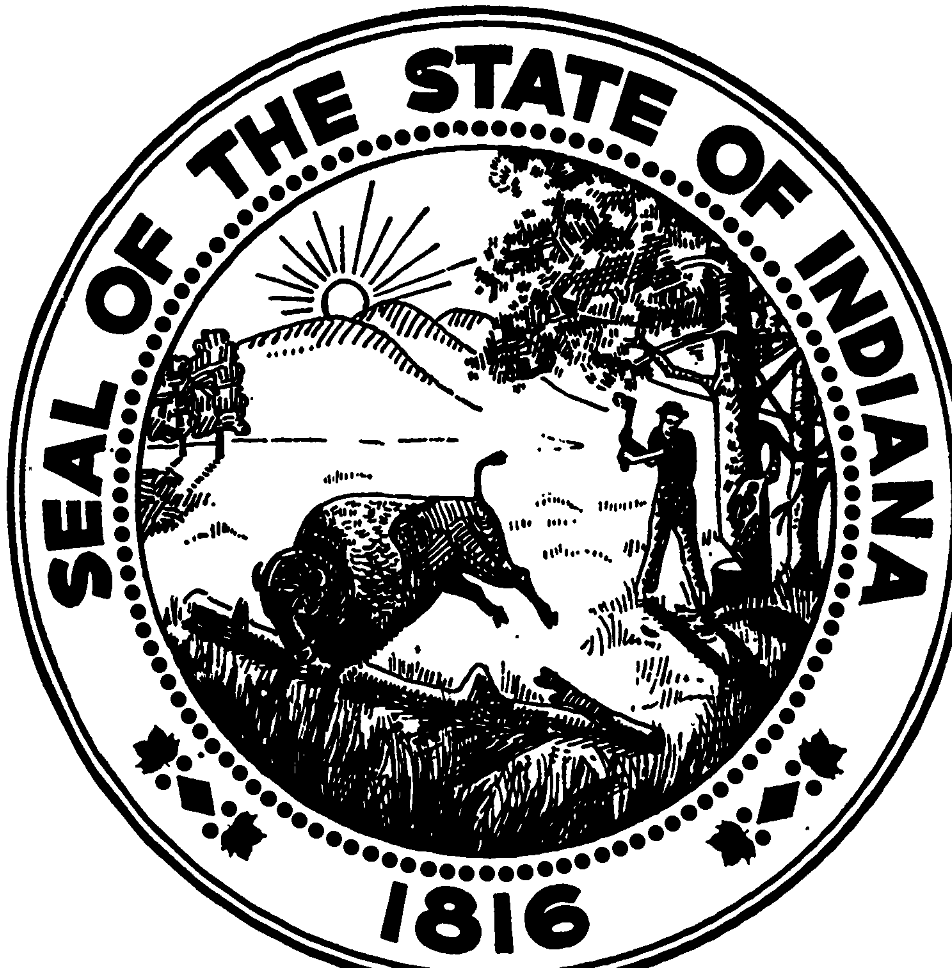
TITLE MATHEMATICS GUIDELINES FOR INDIANA SCHOOLS K-12.
INSTITUTION INDIANA STATE DEPT. OF PUBLIC INSTRUCTION, INDIANAPOLIS.
PUB DATE 69
NOTE 104P.

EDRS PRICE EDRS PRICE MF-$0.50 HC-$5.30
DESCRIPTORS *CURRICULUM DEVELOPMENT, *CURRICULUM EVALUATION, CURRICULUM GUIDES, EDUCATIONAL OBJECTIVES, *ELEMENTARY SCHOOL MATHEMATICS, MATHEMATICS EDUCATION, *SECONDARY SCHOOL MATHEMATICS

ABSTRACT

THIS PUBLICATION IS A MATHEMATICS CURRICULUM GUIDE FOR THE SCHOOLS IN THE STATE OF INDIANA. ITS PURPOSE IS TO ASSIST LOCAL SCHOOL DISTRICTS IN THE EVALUATION OF EXISTING MATHEMATICS CURRICULA AND TO PROMOTE THE DEVELOPMENT OF BETTER MATHEMATICS PROGRAMS AT ALL GRADE LEVELS. THIS GUIDE OFFERS PHILOSOPHIES OF ELEMENTARY AND SECONDARY MATHEMATICS INSTRUCTION AS WELL AS OBJECTIVES IN TEACHING MATHEMATICS. ELEMENTARY PERFORMANCE OBJECTIVES ARE ORGANIZED IN TERMS OF THE UNIFYING CONCEPTS WHICH APPEAR THROUGHOUT THE ELEMENTARY SCHOOL MATHEMATICS PROGRAM. SECONDARY PERFORMANCE OBJECTIVES ARE PRESENTED IN TERMS OF SPECIFIC COURSE OBJECTIVES. A GUIDE FOR TEXTBOOK SELECTION, A CHECKLIST FOR EVALUATING TEXTBOOKS, AND TEACHER AND STUDENT BIBLIOGRAPHIES ARE ALSO INCLUDED. (FL)

MATHEMATICS GUIDELINES FOR INDIANA SCHOOLS K-12



OFFICE OF STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

RICHARD D. WELLS, SUPERINTENDENT

MATHEMATICS GUIDELINES FOR INDIANA SCHOOLS K-12

Published by the
Office of State Superintendent of Public Instruction

Richard D. Wells, State Superintendent
Indianapolis, Indiana
1969

Prepared by the

Indiana State Mathematics Advisory Committee

Sponsored by

Office of the State Superintendent of Public Instruction

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Message From The State Superintendent

This publication has been developed to help implement a coordinated K-12 mathematics program in the schools of Indiana. It is presented to Hoosier teachers with the hope that it will be useful in defining and appraising the mathematics curriculum for all grade levels.

The project was undertaken by a committee composed of elementary teachers, junior high teachers, high school teachers, mathematics supervisors, and university mathematics educators. Some of the people on the committee have had experience on writing committees for major curriculum projects and others have authored textbooks. Many have involved themselves in the preparation of local curriculum projects. All have had wide experience in mathematics education at some level.

The committee's intention has been to provide a structure to help local curriculum committees develop their own local Course of Study. It is not the intention of this guide to be prescriptive. Rather, it is hoped that it will be thought of as a beginning point from which local curriculum groups will eventually be able to construct a truly coordinated and comprehensive mathematics program.

Much work and thought has been devoted to the development of this guide. The Office of the State Superintendent of Public Instruction is indeed grateful to the members of the committee for their efforts in behalf of the boys and girls of this State. Thanks is also due to the many superintendents of local schools and deans of the various universities represented on the committee for releasing their teachers to do this most essential work of curriculum development.

RICHARD D. WELLS
State Superintendent of Public Instruction

Preface

Mathematics Guidelines for Indiana Schools K-12, of 1969, has been prepared to replace Bulletin 212, *Mathematics For Secondary Schools* (A Guide to Minimum Essentials) of 1963. This new publication was constructed by the Indiana State Mathematics Advisory Committee under the sponsorship of the Office of the State Superintendent of Public Instruction. It is meant to be of use to Indiana educators as they perceive the need to appraise their mathematics curriculums.

The Advisory Committee held its first meeting on November 1, 1968. While discussion ranged over a wide variety of topics, the attention of the Committee soon focused on the need for a new state curriculum guide. A steering committee was selected and met on November 20, 1968 to formulate plans. These were submitted to the entire Committee on December 13, 1968. Writing assignments were made and discussion centered on the type of objectives which would appear in the guide. It was decided eventually to couch the objectives in a modified behavioral format. That is, the objectives would specify the behavioral (performance) act but omit stating the conditions of performance or the criteria for acceptable performance. The latter two items could be built into the kinds of objectives that local school districts might construct. These objectives were to form the foundation on which the publication would be constructed.

A "Working Copy" was completed in early March. Revisions were written in March and April. The Working Copy and initial revisions were sent out to reviewers throughout the state for their appraisal and comment. Aided by this most valuable contribution, the Editorial Board reviewed the entire publication in May and early June. Final editorial revision was completed during the last week of June, 1969.

This publication will not supplant local effort toward curriculum renewal. It will not substitute for a well prepared local Course of Study. But, it is expected that most schools of the State will find this publication helpful in the preparation of their own Courses of Study.

Table of Contents

TABLE OF CONTENTS (Continued)

Part I

General

PHILOSOPHY OF ELEMENTARY MATHEMATICS INSTRUCTION

The variety of mathematics programs now available for the elementary grades promises to benefit both teachers and pupils alike. Having been freed from the fixed form that characterized the traditional program, teachers now have a greater opportunity than ever to select the materials and teaching techniques best suited to a child's needs. The basis used for selecting methods and materials is important and is dependent upon the viewpoint of the teacher in regard to his instructional task. Just as mathematics programs have changed, so might certain aspects of the traditional view of instruction also change.

It has been common practice heretofore to consider certain mathematics skills and concepts as being "all-or-nothing" attainments. The failure of some children to achieve the expected level of proficiency has forced the teacher of the next grade to "reteach" these areas, a task that has long been a source of irritation to many teachers. This type of compartmentalization of a course of study should be minimized, if not eliminated. Instead of expecting all children at a given grade level to master in a rather final sense a certain portion of the mathematical training, educators should develop an approach which recognizes the existence of individual growth rates and stresses the continuity of the instructional process.

Perhaps the one factor most essential to the success of the mathematics curriculum is an emphasis on understanding, that is, understanding of mathematics. This emphasis represents a marked shift in the focus of educators' attention from overt behavioral skills and social applications to understandings developed as a child organizes and codifies mathematical ideas. It is no longer deemed sufficient for a teacher to be satisfied with competent computational performance by a child in spite of the obvious necessity for such skill. The need to find a more efficient, more enjoyable, more illuminating method of instruction has led to the consensus that clear and penetrating understanding of certain essential mathematics must precede, but certainly not supplant, the traditional point of emphasis, computation. Certain principles of instruction deserve renewed emphasis. It is agreed that one should traverse from the known to the unknown; from the particular to the general; from the concrete to the abstract. Clearly, children should be encouraged to develop their first concepts of mathematics from their experiences with physical objects.

This approach implies, in many cases, that prior to the introduction of a concept, time should be provided for each child to experiment with physical objects appropriate to the objective. For instance, every child should have the experience of actually combining sets or groups of things as a prelude to the introduction of addition; he should work on an abacus or place-value device prior to any contact with sophisticated positional notation; he should have "pre-number" experiences, such as learning to compare the numerousness of groups through the process of one-to-one matching rather than counting in the traditional fashion.

Because understanding is an ambiguous term and because a teacher must make as clear as possible the relationship between former objectives and present points of emphasis, it may be well to consider just a few of the many concepts that are especially important to clarify and develop from the earliest stages.

For the foreseeable future, the study of number systems will continue to be the essence of the mathematics program. Involved in this study are the collections of various kinds of number ideas, such as natural numbers, together with the operations defined on these number ideas and the properties of these operations (for example, commutative, associative, distributive properties). Obviously, the development of an understanding of such ideas must take place over an extended period of time and at considerably different rates for different children. There is no such thing as complete understanding of any particular number system by a child. Rather, teachers at each level should ascertain that the instruction is directed toward deepening and extending the broad mathematical streams.

As a child's grasp of mathematics grows, he must be guided toward the acquisition of the broad and essential concepts which tie together seemingly disconnected particulars into a coherent general structure. The same operations are encountered in several different contexts in the grades. Each particular use really represents a different operation.

Natural numbers, integers, and rational numbers each combine somewhat differently under a given operation. But rather than invent new symbols to represent the changing definitions of an operation as it applies to the various number systems, one uses the same symbol throughout. In this situation, as in many similar ones, children must be taught to interpret symbols in context. Only in this way will mathematics shorthand serve the important purpose of clarifying and improving communication. Furthermore, the child should be led to understand that although the specific interpretation of an operation differs from one system to the next, a structural unity still remains which is founded on the commutative, associative, and distributive properties as they pertain to the operations. Because of this structural unity, one can rely on the context in which a symbol is used to determine its meaning, rather than create new symbols for each particular variation of a concept.

Again, the need for techniques which lead the child to discover ideas through his experiences with physical objects must be stressed. Furthermore, attention must be given to rather subtle ideas that may have been overlooked or thought too obvious to mention in the past. For instance, when a child combines two sets of objects into a single collection, he must come to the realization that the abstract property of numerousness is conserved; that there are exactly as many objects after as there were before the two groups were joined. As fundamental as such an assumption is to understanding the addition operation, it is by no means obvious to all beginners.

Another matter often treated too lightly involves the freedoms or restrictions which may or may not exist for a given operation. When one matches the elements of a set with the word names of the ordinal numbers, he must know that the order in which the objects are matched to the numerals will not affect the count. On the other hand, cases exist in which the order of presentation of information is important. One cannot mix up numerator and denominator with impunity or name the coordinates of a point by whim. The basis for free choice, where it exists, and the reasons for restrictions, as they occur, must be made clear if understanding is to be achieved. Such matters are rarely as obvious to children as they seem to adults.

To sum up, children should come to realize that mathematics is a logical system which exists as a human invention, formulated, enriched, extended, and revised in response to the twin needs to perfect it as a logical structure and to use it as a convenient method of describing certain aspects of nature as seen by man.

Educators should avoid giving undue emphasis to any single aspect of the total mathematics program. The idea of sets, for instance, should not be glorified beyond its usefulness in contributing to the attainment of the broad goals of the program. Similarly, it is unwise to go to the extreme of downgrading the importance of computational proficiency to the point where long range goals are placed in jeopardy. Furthermore, impressive terms and symbols must never interfere with a child's understanding of the concepts which the words or symbols represent. Clarity of communication is the chief purpose of mathematical language and symbolism. If a particular term or symbol does not serve the cause of clarity, then it should not be used. Children, of course, must eventually learn how to use the language effectively and understand it in context. However, teachers should exercise great care in working toward such a goal and should avoid any proliferation of symbols or premature verbalization that will hinder goal attainment.

One of the greatest challenges for the elementary teacher is the development of appropriate real life problems which meaningfully involve learners. Classroom teachers recognize the limitations of textbooks in providing such verbal problems. Many problems fail to challenge children; they fail to represent a world which is real to children; and too often they fail to contribute to the child's skill in problem solving situations outside the textbook. Notwithstanding these limitations, textbook verbal problems will continue to be prominent in most elementary classrooms until individual teachers identify other ways of meeting the problem solving obligation.

There is increasing evidence that through experimental programs and through the efforts of individual classroom teachers, improved ways of meeting this obligation are being developed. Much is being heard about the importance of relating mathematics to science, about units of study in mathematics, of enrichment activities in mathematics, about the study of the history of mathematics, and about children creating their own mathematics problems. Instead of stressing the social development of the child, these efforts emphasize the structure of mathematics through

problem solving experiences. These activities are hopeful signs pointing to a greater stress being placed, in the near future, on the importance of meaningfully-structured problem solving experiences as a part of the elementary mathematics program.

If mathematics instruction is viewed as a process of initiating understanding and of carefully nurturing this understanding as the child matures, it will then be necessary to discover effective techniques for accommodating the widely differing rates at which children develop. Implicit in this statement is the need for schools and teachers to recognize that some children may not be able to grow substantially or may seem to terminate their potential for growth at some point along the way. In such cases, provision should be made for experiences most appropriate to the child's welfare.

In summary, mathematics instruction should be viewed as a continuous effort to develop in the child a knowledge of mathematics that is characterized by its depth and connectedness. To the extent possible, the child should be encouraged to experiment with the objects of his environment. Thus prepared, he may be led to the invention or discovery of those ideas which provide both a broad basis for further exploration and a sense of delight in a well-founded mastery of the subject. The task is a challenging one and deserves much effort. To this end, the teacher should do everything possible to see that instruction is well-planned and is provided on a regular basis. As an additional, but essential measure, cooperative action such as inservice education should be taken by school personnel to develop in the teaching staff a view of instruction appropriate to present day needs and opportunities.

PHILOSOPHY OF SECONDARY MATHEMATICS INSTRUCTION

The language of mathematics permeates the fabric of our society, and to be ignorant of its signs and symbols, its processes, and its methods is to be unable to read with insight and understanding the publications which come to our homes. Modern man perceives the world in which we live as incurably mathematical, and modern culture in a large measure derives its substance and its texture from the science of mathematics. Mathematics is the medium by which the human genius records its observations upon the structure and behavior of nature and from which it draws the power to discern the underlying order. It supplies man, in short, with the basis for broadening his understanding and for bending his knowledge to useful ends.

The whole matter of the character of mathematics and its role as a human enterprise is, unfortunately, all too little understood, despite the fact that in a literate society such as ours everyone is at some time exposed to some mathematical instruction. The great majority of people, even those who are otherwise well informed, are utterly unaware of both the vastness and the diversity of mathematical thought and of its living, dynamic character. They do not realize that mathematics is the central directive agent of the whole human quest for quantitative understanding of the environment; that it is in command at all points in man's efforts to apply nature to his advantage; that it is the medium through which chaos and superstition are banished; and that it is the backbone not of one science or several, but of all sciences.

Mathematics is not a set of unrelated and isolated topics. It is a system of ideas unified by such concepts as number, measurement, relationship, operation, symbolism, and proof which relate to other fields of learning and which grow in meaning and significance for the student as his study continues. To nourish the growth of these concepts, to develop their relation to the problem-solving process, and to open channels of investigation will provide the academic and vocational flexibility so essential in guiding the educational theories of students.

The call for more effective teaching of mathematics is one of the most important challenges of today. From this, every mathematics teacher surely can draw a measure of gratification, for it manifestly emphasizes the importance of the mathematics teacher's job. Personal and professional satisfaction rest upon that but so also does larger responsibility. In countries under the sway of dictatorial rule, pupil enrollments in mathematics classes or, for that matter, in any classes can be raised by edict and maintained by regimentation. Those expedients are not ours; we neither can nor wish to use them since we have no desire to deny any individual's right to choose in matters of his own career. Persuasion and guidance are less prompt than compulsion and are more demanding of patience and devotion. Nevertheless, they seem to us to be the preferable means and to be more certain of human results.

As a social force mathematics has changed with the times and is continually changing. Even in the very recent past, mathematics has generated an abundance of fruitful ideas; on the basis of these ideas, new disciplines have been born and older ones reconstructed. New concepts and other recent developments have been extensive and revolutionary.

We may ask ourselves "What of tomorrow?" In no way can one visualize tomorrow's life, tomorrow's problems, or tomorrow's instruments of solution. It is no longer possible to prepare young people through experiences that relate solely to their immediate environment. Tradition must give way so that the unlimited resources of the mind are released to confront the unknown with imaginative plans. The docile, uninterrupting, information-absorbing student is a figure of the past. He has become transformed into a participating being, thriving on challenge and demanding reasonable, adequate explanations.

Among the goals of the secondary school are to nourish cultural enlightenment and to furnish the student with a broad spectrum of skills and concepts. To hinder the student from traversing varied subject fields and potential interests would frustrate the achievement of these goals. The experience of relating disciplines to one another and to the whole of learning should assist the student in realizing his potential.

Secondary school mathematics extends beyond a concern with manipulative procedures. The curriculum must afford students opportunities for working with basic concepts of arithmetic, algebra, and geometry. But more than this, it must provide for the construction of symbolic and abstract models. In summary, student discovery of the internal structure and logical cohesion of mathematical systems is an important new dimension in the mathematics curriculum.

OBJECTIVES IN TEACHING MATHEMATICS

Current thinking regarding curriculum construction seems to center around four major components:

1. What shall we teach?
2. In what sequence shall we teach?
3. What methods or system of instruction can best help the learner achieve what has been considered worthy of being taught.
4. What criteria determine whether or not the learner has actually learned?

This guide provides a possible choice of topics and a possible sequence of mathematics instruction based upon the best available professional opinion of what is relevant, meaningful, and necessary for the development of mathematical understanding for participation in our society. There are no references to teaching procedures because learners and teachers appear to have wide variability in styles of learning and teaching effectively. The literature regarding methods of attaining particular outcomes continues to grow, and the responsibility of professionals in mathematics education to remain abreast is not decreasing.

Hopefully the day has past in mathematics education when the teacher opened the book on the first day of school and started teaching "about mathematics." Every day requires consideration by the teacher of not only what has been learned by pupils in each class but, just as important, what types of performance or behavior indicate that learning has taken place. In other words, consideration must be given to objectives in terms of performance. What is it that a pupil says, writes, or does that indicates that he has learned? After this has been determined, the methods of instruction may be chosen from the myriad of possibilities to elicit desired performance.

Suppose a teacher is concerned with teaching students about square roots. He wants students to know that a square root of a number is the factor of the number which, when multiplied by itself, produces the number. He would also like students to know about the dividing and averaging method of finding the square root of a four-digit positive whole number rounded to the nearest hundredth. Most mathematics teachers would be unsatisfied that the student knows about square roots until he can give correct responses to questions or exercises regarding square roots. He might want the student to be able to state the square root of three four-digit whole numbers by the dividing and averaging method. He would undoubtedly want students to distinguish between the square and square root of a number.

A long discussion could ensue here, and probably should, regarding whether or not these are adequate performance objectives indicating evidence of learning. Such discussion probably would produce better performance objectives and the corresponding means of attainment. This then represents a useful purpose of this guide: namely, to stimulate thinking of teachers and curriculum developers to concentrate on learner behavior or performances that indicate that learning has taken place. The next step would be to consider the means to elicit those performances through carefully contrived, artful, and creative learning experiences.

The authors of this guide recognize that at the present time there is controversy over both the terminology and the definitions of terms relating to the statement of educational objectives. Feeling that a lengthy discussion over the merits of any particular system of terminology or definitions would violate the purpose of this Guide, the authors have chosen arbitrarily to use the terms *global objectives*, *performance objectives*, and *instructional objectives*.

Global Objectives

It is recognized that the execution of mathematics education in the schools may (and perhaps should) exceed any set of stated global objectives. While it is essential that students develop basic skills and problem-solving techniques and that they increase their mathematical literacy and their concept of the logical structure of mathematical systems, mathematics education must do more. It must nourish the growth of those apperceptions, insights, and attitudes in students which enable them to distinguish fact from fiction, testimony from testimonials, relevant from irrelevant materials, empirical data from proven theorems and stated hypotheses. It should stimulate a spirit of inquiry and curiosity which will impel the student to learn new mathematics by independent

study. It must effectively relate mathematics to the large body of knowledge in the other disciplines (e.g. science, social studies).

Still, the Committee preparing this Guide finds it helpful to state certain basic mathematical needs of the students. The following list of global objectives is proposed:

1. The student should come to comprehend how mathematics contributes to the analysis of events that occur in the physical world.
2. He needs to be aware of the important role mathematics plays in the evolution of his civilization.
3. He needs to use the words and symbols of mathematics with precision so that he will be able to communicate mathematical ideas correctly and clearly to others.
4. He needs to develop a mathematical literacy which will enable him to make wise decisions as a producer and/or consumer of products and services.
5. He needs to acquire proficiency in the use of common mathematical processes.
6. He needs to become sufficiently prepared at each level for further study of mathematical concepts and operations.
7. He needs to acquire habits of orderliness, competence in logical argument, and an appreciation of the nature of mathematics as a logical, deductive system built on a structure of unifying concepts.

Performance Objectives

The term *performance objective* is used to designate objectives at that middle level of educational goals which identify the desired behavioral action to be measured without specifying either the conditions under which the act will occur or the criteria for acceptable performance. This Guide is written giving sets of performance objectives by strands (for the elementary grades) and by course (for the grades 9-12).

Instructional Objectives

The term *instructional objectives* has been assigned to the operational level of objective which *does* specify the conditions and criteria for acceptable performance. Instructional objectives are what local teachers and curriculum planners should derive for their own curriculum guides and course offerings from the performance objectives suggested in this Guide. Each performance objective listed here may result in an array of instructional objectives which explore the various aspects of, or the subskills underlying, the behavior specified in the performance objectives.

An Example:

The relationship between the various objectives described in this section is indicated by the example and diagram that follow.

Global objective: The student should come to comprehend how mathematics contributes to the analysis of events that occur in the physical world.

Performance objective (elementary): The student should be able to find the area of a rectangular region.

Performance objective (secondary): The student should be able to find solution sets to selected systems of equations.

Instructional objective (elementary): The student should be able to correctly name the measure of the area of a rectangular shaped floor which is $23\frac{1}{2}$ feet long by 14 feet wide.

Instructional objective (secondary): After in-

Global Objective 1.
Global Objective 2.
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{ Performance Objective 1.
Performance Objective 2.
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etc.

{ Instructional Objective 1.
Instructional Objective 2.
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struction and experience with Cramer's Rule and when presented with a set of ten systems of three equations in three variables, the student will find solution sets for at least seven of them.

The reader will note that as the level of objective proceeds from global to performance and from performance to instructional, the objective becomes more specific. This Guide will fulfill its intended function only as local curriculum planners and mathematics teachers translate its performance objectives into more specific instructional objectives that can be applied immediately to the development of courses, units, and lessons for classroom use.

TESTING AND EVALUATION

Assuming that the purpose of teaching students is to effect intended changes in their behavior, it follows that measuring instruments should be devised which will evaluate their progress toward those changes. These instruments will normally call for responses which are written, verbal, or kinesthetic; yet the responses will not necessarily be immediate.

Essential to the construction of measuring instruments are the determination of the intended changes and the definition of acts of behavior which will disclose the extent to which the changes have occurred. Essential to the interpretation of the data obtained by the measuring instrument are the qualifications which have been built into the specific objectives of the Course of Study. That is, test items should emerge nearly intact if the objectives and the conditions for performance have been stated clearly. If each of the objectives carries its criteria for acceptable performance, then inevitably the quantitative part of the evaluation of performance becomes nearly automatic.

In evaluating students, we can list three levels of evaluation through observation of students' performance:

1. The measurable observations of students' performance such as "getting the right answers."
2. Observations of students' performance indicating that they understand the process but perhaps made an error in the computation.
3. Observations of the students' appreciation (through discussion, interaction, etc.) of the relationships between or beauty in mathematical ideas.

The qualitative, subjective part of evaluation poses a real problem. This part of the evaluation, however, must be brought to bear in the evaluation of student performance. It must embrace the cumulative total situation in which the student finds himself rather than simply the arithmetic average of his overt performances.

Schools may find this publication helpful in preparing measuring instruments. However, the objectives listed in this publication have not included the qualifying features, i.e., the conditions of performance and criteria of acceptable performance. It was felt by the Advisory Committee that it would be inappropriate for the State to suggest qualifying conditions for pupil performance. School staffs, together with students and parents, are in an excellent position to determine more extensive objectives which will include these qualifying features. Then, evaluation will become more meaningful through shared efforts and shared responsibilities.

If a school wishes to purchase commercially prepared tests, the following is a partial list of sources:

Bureau of Educational Measurements, Kansas State Teachers College, Emporia, Kansas 66801.

California Test Bureau, 5916 Hollywood Boulevard, Los Angeles, California 90028.

Bureau of Educational Research and Service, State University of Iowa, Iowa City, Iowa 52240.

C. A. Gregory Company, 345 Calhoun Street, Cincinnati, Ohio 45221.

Educational Test Bureau, 720 Washington Ave., S.E., Minneapolis, Minnesota 55414.

Educational Testing Service, 20 Nassau Street, Princeton, New Jersey 08540.

Public School Publishing Company, 345 Calhoun Street, Cincinnati, Ohio 45221.

Science Research Associates, 9880 West 10th Street, #36-7, Indianapolis, Indiana 46234.

Another publication on testing available from National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D.C. 20036, is *Mathematics Tests Available in the United States, 1968 Edition.*

While mathematics educators must be concerned with evaluation of pupil performance, attention should also be given to the evaluation of the mathematics curriculum itself. Not only is it necessary to know how well the intended objectives are being realized, but also it is important to consider whether the objectives are appropriate for students' needs.

Contemplation of the following questions may disclose the appropriateness of the school's curriculum:

a. To what extent do students find the mathematics courses interesting and profitable?

b. Is the mathematics curriculum dedicated toward fulfilling the needs of the entire student body? How well are individual differences provided for?

c. How relevant is the mathematics to the times?

d. What is the balance between conceptualization and computational skills?

e. To what extent are unifying concepts and structure made an integral part of each course and topic?

PROVIDING FOR INDIVIDUAL DIFFERENCES

The authors of this guide recognize the importance of providing for differences in intelligence, achievement, maturity, learning rate, emotional and social behavior, physical condition and interest. Having said this, however, it is acknowledged that any one suggestion about providing for individual differences will probably fall short of application for a specific case. It is the classroom teacher who still must choose from a variety of instructional techniques and materials the means for providing an exciting, attractive learning situation for each student.

What is essential to the provision for individual differences is a firm commitment to the idea of implementing a truly individualized instructional program which will enable a larger percentage of students to have successful learning experiences—successful in the sense that they have achieved intended goals. While the road to providing this kind of program is not entirely clear at this time, still the authors of this guide take the view that a sensitive teacher, attuned to the emotional and intellectual needs of his students, and dedicated to the educational welfare of each student in his class, can help each student to have successful learning experiences. Further, many of these successes will occur within the structure of the traditional classroom setting if teachers and students will plan their experiences to take maximum advantage of the many instructional aids available to schools today. Some schools also are attempting to help teachers provide for student differences by organizational variations such as middle school, nongraded school, flexible, modular scheduling, etc.

There are many ways of providing for individual differences; however, these should be considered valid in many cases only as the presence of individual differences is determined. It may be that groupings of students are valid for some particular topic of instruction but completely irrelevant for the learning of other topics.

Teachers can provide for individual differences in teaching certain topics in their classrooms. Among the techniques for doing this are:

1. Grouping students within the clasroom according to their level of mathematical development and presenting to each group subject matter which fits that group in its scope and/or depth.
2. Grouping students from several classrooms for large group presentations by teachers cooperating and exploiting each other's strengths.
3. Grouping students by teams of two or more in which the leadership role is assumed by the students under the guidance of the teacher.
4. Making flexible and optional assignments which are, wherever possible, related to the life experiences of the students.
5. Supplying appropriate alternate materials such as: a. programmed materials, b. prepared tapes, c. film loops and motion pictures, d. games and activities, e. calculators, f. abacus, g. problems to solve from other academic and vocational areas.

Other suggested activities for differentiating instruction and caring for individual differences are:

1. Individual and group reports on topics investigated.
2. Application of mathematics in solving community problems.
3. Use of supplementary textbooks.
4. Mathematics Clubs.
5. Mathematics exhibits and contests.
6. Constructing models and other visual aids.

The following steps are offered for your use in your efforts to individualize instruction:

1. Specify the objectives of the learning task in terms of student behaviors.
2. Select or devise instruments and procedures to measure the attainment of the objectives.
3. Pre-test to determine which objectives the student has already mastered.
4. Diagnose each student's characteristics as a learner in relation to the task.
5. Prescribe for each student learning activities that will lead to mastery of the task.
6. Select learning materials and equipment the student will require.
7. Program oneself as teacher to guide the student in terms of their prescriptions.
8. Individualize the instructional activities and revise the prescriptions as needed.
9. Keep adequate records of pupil performance and progress.

INSTRUCTIONAL AIDS

Instruction in mathematics, as in all subjects, can be made more effective through the proper use of suitable teaching aids. Mathematics is primarily concerned with ideas and abstractions, but an understanding of the ideas often grows from concrete experiences. This is true from sets of counting sticks in kindergarten to devices which illustrate rates of change in calculus.

In recent years the educational market has been flooded with many new and useful aids to instruction. It would not be possible to name and illustrate them all in this guide. Catalogs are often available which present a wealth of ideas. Many teachers find it is more effective, when possible, to have the pupils make their own devices than to have commercially made ones.

Instructional aids may be classified in the following ways:

1. Major pieces of equipment include chalkboard, tackboard, projectors, recorders, television receivers, computer terminals, and so on.
2. Devices or models either for demonstration or pupil use. These normally have application to specific topics.
3. Audio visual supplies such as films, filmstrips, transparencies, etc.
4. Printed materials: remedial, supplementary or enrichment.

Instructional aids can contribute to effective teaching techniques in many ways. The following uses are cited:

1. Help students discover concepts and principles
2. Increase motivation
3. Enrich instruction
4. Introduce new concepts
5. Simulate life situations
6. Illustrate applications
7. Challenge pupils to solve problems
8. Provide visual or tactual reinforcement of ideas
9. Provide needed practice
10. Promote variety in activities

Teachers who wish to keep up-to-date on the latest developments in the field of mathematics teaching aids are encouraged to attend meetings of the National Council of Teachers of Mathematics and other professional organizations. The exhibit area at such meetings provides a display of virtually all materials and devices on the market. It is more satisfying to examine these aids first hand than to look at pictures in a catalog. (This is evidence that aids in the classroom are more valuable than pictures and are far more valuable than words alone.)

A partial listing of sources of instructional materials and the names of the Indiana representatives of these firms appear each year in the back of the Indiana School Directory published by the Office of the State Superintendent of Public Instruction. A copy of this Directory is sent to each superintendent and principal.

Part II

Elementary Performance Objectives

MATHEMATICS ACCORDING TO STRANDS

The Elementary Section of this publication is organized in terms of mathematical strands. These strands might be listed in many different ways and are, as a matter of fact, listed differently by different mathematics educators. The eight strands by which the elementary school program is organized in this publication are not necessarily the only strands which might be listed. These strands, however, in the minds of the writers of this publication, are sufficiently distinct and important to organize the content of an elementary school mathematics program.

A strand can be thought of as a unifying thread that is woven through the elementary school program from grade one (informal activities in Kindergarten) up through grade eight. The concept in a given strand which is developed in the first grade is added to and implemented in the second grade, in the third grade, and all the way through the eighth grade. In looking at a mathematics program from this point of view, it is sometimes easier to see the organization and structure with which a child grows in mathematical knowledge. The eight strands that the authors of this publication have selected are: *Sets—Numbers—Numerals, Numeration, Operations and Properties, Mathematical Sentences, Geometry, Measurement, Graphing and Functions*, and *Probability and Statistics*.

The careful reader will note that neither Computation nor Problem Solving nor Logical Thinking are listed as strands and may question the reason and wisdom in not including them. The authors of this publication see that Problem Solving, Computations, Estimations, Logical Thinking are competencies to be achieved by pupils and are not inherently a part of the content of mathematics. Thus, Problem Solving, Computation, Logical Thinking, and so forth all are competencies that a pupil ought to acquire as he is studying about Measurement, Geometry, or Operation and Properties, or any mathematical strand. In other words, as the mathematics content is taught, teachers should be aware that the pupils should be gaining in their Problem Solving proficiency, in their Computation proficiency, in their Estimation proficiency, and in their Logical Thinking proficiency.

The "pupil strands" of Problem Solving, Computation, Estimation, Logical Thinking, etc. should be developed while teaching the mathematical content. The fact that these pupil proficiencies are not itemized is *not* an indication that they are unimportant. In fact, just the opposite. It is really the pupil's acquisition of these competencies that a teacher should be continually aware of as he is developing the mathematical topics with the pupil. If the content program is well ordered and organized and the teacher is aware of the competencies that ought to be acquired by pupils, there is a greater chance for learning than if the distinction between the content strands and the pupil competencies is not made.

A brief description of each of the strands follows:

Sets—Numbers—Numerals

This strand deals with the study of numbers and how they are named. (It does not include the system such as the decimal system of recording such numerals.) The strand deals with whole numbers, rational numbers, integers, as well as prime numbers, composite numbers, and, in general, other elements of number theory. It also deals with sets, their relationship to numbers, and with elementary logic.

Numeration

This strand has to do with man's way of recording numbers in a systematic form. The decimal numeration system is carefully studied. Other numeration systems are referred to for comparative and cultural properties.

Operations and Properties

This strand deals with how we use numbers in addition, subtraction, multiplication, and division. This strand also examines the structure of numbers under certain operations by introduction to the properties of certain operations.

Mathematical Sentences

This strand encompasses man's method of translating verbal and physical situations into mathematical language. The relationships of equality and inequality are studied in the form of open sentences, as well as statements. Place holders

and solution sets are found, as well as interpreted.

Geometry

This strand examines the elements of spatial relationships. It examines the particular arrangements of points, lines, planes, and spaces and their geometric shapes as related to these concepts. Metric geometry deals with the measures of these figures. Non-metric geometry deals with the properties and relationships among these figures.

Measurement

This strand treats of man's method of recording measure of all measurable quantity; time, weight, temperature, length, etc. can all be measured. This strand overlaps with the metric geometry strand in measures of length, area, and volume.

Graphing and Functions

This strand deals with man's way of picturing numbers on a line, pairs of numbers on a plane (grid), and triples of numbers on an x-y-z coordinate system. This strand also deals with the special relationship, function, of one set of numbers to another. An example of such a function is a ratio (miles per hour, apples per cents) and per cent.

Probability and Statistics

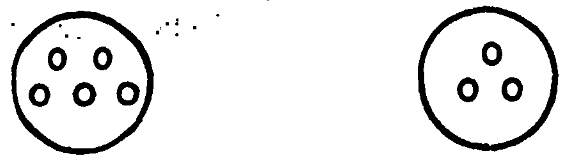
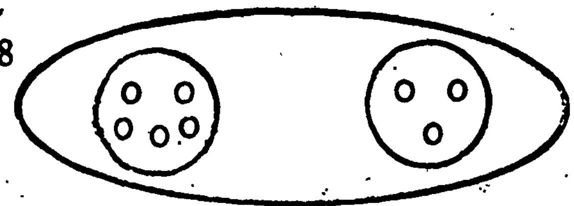
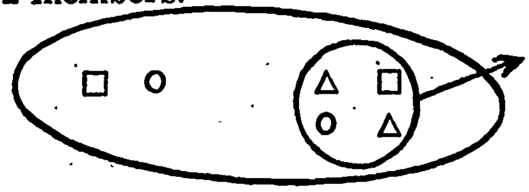
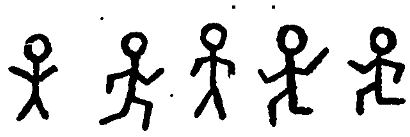
This strand deals with collecting and interpreting data. It deals with the chance of something happening, and with the fundamental way of counting for probability.

Sets—Numbers—Numerals: Performance Objectives

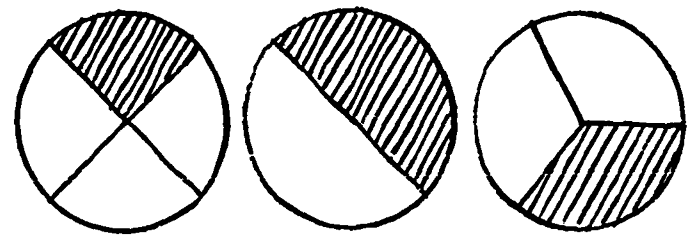
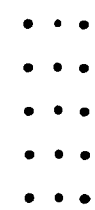
Suggested Grade Levels

	K	1	2	3	4	5	6	7	8
Items	1 to 12	1 to 24	14 to 27	20 to 32	28 to 40	34 to 44	35 to 49	45 to 55	45 to 55

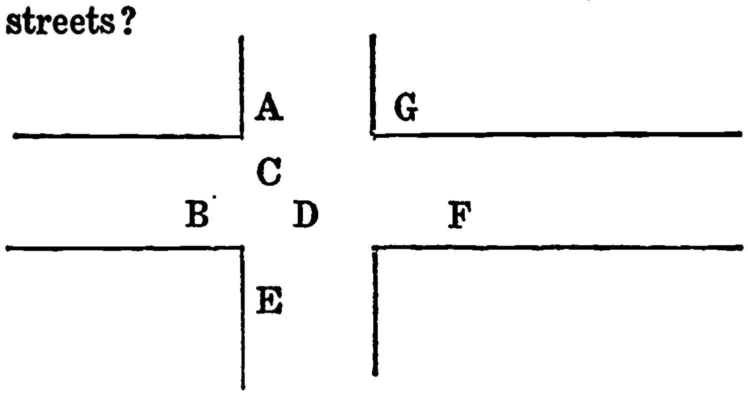
Objectives	Examples
1. Form a one-to-one correspondence between two sets of objects which have the same number of elements.	1. Child gets a book for each child in his group.
2. Compare two sets of objects and determine which set has the greater number of objects.	2. Child finds that there are more children than balls, as he passes out balls at play time.
3. Associate appropriate spoken number names with sets, for zero through five.	3. Child tells how many are in a set; or exhibits a set when asked to "give me three blocks."
4. Place sets of objects in order according to the number of objects in each set, from the least to the greatest.	4. Child arranges sets of blocks in order: 
5. Say the number names in order.	5. Child sees sets arranged in order, from least to greatest, and says the number names for the sets in order.
6. Associate number names with sets containing six to nine objects. Place these sets in order and see subsets containing one to five objects in these sets.	6. Child exhibits many ways to group seven objects: three and four; two and two and two and one; five and two; etc.
7. Count through ten placing number names, beginning with one, in one-to-one correspondence with objects being counted.	7. Count the number of objects in a set of ten.
8. Show sets of ten objects.	8. Use finger plays involving ten fingers. When taking attendance match each child with a bead on the counting frame; children decide how many rows of ten beads and how many more are needed.
9. Use equivalent fractions concept in experiences.	9. $\frac{1}{2}$ of a pie is equivalent to $\frac{2}{4}$ of a pie.
10. Use ordinal numbers.	10. Child finds the second book on the third shelf.
11. Give examples which relate concepts of integers to experiences.	11. Children discuss differences in temperature as recorded on thermometer.

Objectives	Examples
12. Infer a relationship of greater than, equal to, or less than between two numbers after telling whether one set has as many as, fewer than, or more than another set.	12.  5 ? 3 (? is replaced by "is greater than," "is less than," or "is equal to.")
13. Order a set of numbers (0 to 99) from the smallest to the largest.	13. Order the set of numbers {21, 12, 5, 81, 19} from the least to greatest. $5 < 12 < 19 < 21 < 81$.
14. Name the number immediately before and immediately after any number (1 to 99).	14. Name the number that comes immediately before and immediately after: ____, 70, ____ ____, 19, ____
15. Name the number(s) between any two numbers (0 to 100).	15. Name the number(s) between: 16, ________, 18 48, ________, 51
16. Illustrate, with drawings or aids, the relationship between the joining (union) of two disjoint sets and addition of numbers.	16. A set of 5 and a set of 3 joined results in a set of 8. $5 + 3 = 8$ 
17. Illustrate, with drawings or aids, the relationship between the separating of a set into two subsets and the subtraction of numbers.	17. 4 members removed from a set of 6 results in a subset of 2 members. $6 - 4 = 2$ 
18. Match the ordinal number names with an ordered set containing up to ten elements.	18. Point to the third boy from the left: 
19. List some subsets for a given set.	19. List five subsets of {○, □, △}.
20. Give several names for the same number.	20. Give several names for 15 such as $7 + 8$, 3×5, $20 - 5$, or $5 + 5 + 5$.
21. Order a set of numbers (0 to 999) from least to greatest.	21. List the set of numbers {0, 795, 54, 28, 984, 231, 321} in order from the least to the greatest.
22. Distinguish between even and odd cardinal numbers.	22. What digits might be in the ones place in a base ten numeral representing an even number.

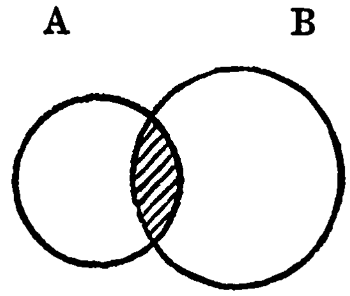
Sets—Numbers—Numerals: Performance Objectives (Continued)

Objectives	Examples
23. Give examples which relate fractions to parts of a whole.	23. Match the shaded part of each diagram with the fraction: $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{2}$ 
24. Give examples which relate equivalent fractions to experiences.	24. One book for each 2 children, or, 2 books for each 4 children, or, 3 books for each 6 children, or, . . . is represented by: $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$
25. Illustrate a multiplication example with sets (in arrays).	25. Show, using an array of dots, that $5 \times 3 = 15$ 
26. Illustrate a multiplication example with product set.	26. With 5 cups of different colors, and 3 saucers of different colors, show all possible cup-saucer combinations.
27. Illustrate a division example using sets.	27. Into how many groups of four can twenty dots be partitioned? 
28. List a set of fractions equivalent to a given fraction.	28. List five fractions equivalent to each of the fractions: $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{5}{2}$ $\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$ $\frac{4}{16}$ $\frac{5}{20}$ $\frac{6}{24}$ $\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$ $\frac{5}{15}$ $\frac{6}{18}$ $\frac{5}{2}$ $\frac{10}{4}$ $\frac{15}{6}$ $\frac{20}{8}$ $\frac{25}{10}$ $\frac{30}{12}$
29. Order a set of rational numbers.	29. Put the set of numbers $\{\frac{3}{4}, \frac{2}{3}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}\}$ in order from least to greatest.
30. List the first eleven multiples of 2, 3, 4, 5, 6, 7, 8 and 9.	30. Set of multiples for 6: {0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60} (Note: 0 is a multiple of every whole number.)
31. Predict "evenness" or "oddness" of the sum, difference or product of two numbers.	31. Will the sum of $17 + 27$ (two odd numbers) be even or odd?
32. Name and list the subsets of a given set.	32. Name and list the subsets of $A = \{a, b, c\}$

Objectives	Examples
33. Determine the prime factorization for a composite number no greater than 40.	33. $36 = 2 \times 2 \times 3 \times 3$
34. Find the common factors of two natural numbers.	34. The common factors of 10 and 12 are 1 and 2. The common factors of 6 and 30 are 1, 2, 3 and 6.
35. Write numerals for a set of fractions equivalent to any given fraction.	35. Write the numerals for three other fractions equivalent to $\frac{1}{3}$. The set of fractions equivalent to $\frac{1}{3}$ is $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots\}$
36. Express any composite number as the product of primes.	36. 60 expressed as the product of primes is $2 \times 2 \times 3 \times 5$.
37. Find the least common multiple of two natural numbers.	37. The least common multiple of 6 and 9 is 18.
38. Find the greatest common factor of two natural numbers.	38. The greatest common factor of 30 and 40 is 10.
39. Give examples which relate positive and negative numbers to realistic situations.	39. Pairs of numbers relate to temperature changes, and differences between positions or scores.
40. Describe the union of two sets.	40. The diagram below shows two city streets. Which letters are in the union of the two streets?



41. Describe the intersection of two sets.	41. Which letters in the diagram above are in the intersection of the two streets?
42. Illustrate the union and intersection of any two sets with Venn diagrams.	42. In the diagram below, Set A is represented by the circle on the left and Set B by the circle on the right. Shade in the region which represents the intersection of the two sets.



Sets—Numbers—Numerals: Performance Objectives (Continued)

Objectives	Examples
43. Express any rational number in decimal notation.	43. Represent $\frac{1}{8}$ and $\frac{1}{3}$ using decimal notation.
44. Give an example of a set, subset, union, intersection, and complement for a given universe.	44. Given $A = \{\text{odd numbers}\}$ $B = \{\text{multiples of } 5\}$ What is $A \cup B$ and $A \cap B$?
45. Name the additive inverse of any integer.	45. The additive inverse of 3 is –3; the additive inverse of –29 is 29.
46. Order any set of integers from the least to the greatest.	46. Order this set of integers from least to greatest. $\{1, -5, 6, 2, -19, 28\}$
47. State the difference between the decimal notation for a rational number and the decimal notation for an irrational number.	47. Which decimal represents a rational number 56.23 or 1.01001000100001 . . . ? How do you recognize a rational or irrational number in decimal form?
48. Use the terms subset, union, and intersection correctly.	48. An angle is the union of two rays with a common endpoint. If two planes intersect, their intersection is a line.
49. Write the prime factorization of any composite number.	49. The prime factorization of 36 is $36 = 2 \times 2 \times 3 \times 3$. $315 = 3 \times 3 \times 5 \times 7$ $248 = 2 \times 2 \times 2 \times 31$
50. List the prime numbers that exist in some interval.	50. The prime numbers between 30 and 50 are 31, 37, 41, 43, and 47.
51. Find the greatest common factor for any pair of numbers.	51. The greatest common factor of 24 and 36 is 12. The greatest common factor of 45 and 75 is 15.
52. Find the least common multiple for any pair of numbers.	52. The least common multiple of 15 and 25 is 75. The least common multiple of 45 and 75 is 225.
53. List a given set of positive rational numbers in order from the smallest to the largest.	53. $\frac{1}{6} < \frac{2}{5} < \frac{1}{2} < \frac{4}{7} < \frac{8}{9} < \frac{10}{9}$ $.003 < .0035 < .004 < .1 < 1.2$
54. Find a rational approximation for an irrational number such as $\sqrt{2}$, $\sqrt{3}$, etc. to the nearest hundredth.	54. $\sqrt{2}$ is approximately 1.41 $\sqrt{3}$ is approximately 1.73
55. Find the Cartesian product of two finite sets.	55. If $A = \{a, b, c\}$ and $B = \{1, 2\}$ then $A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

Numeration: Performance Objectives

Suggested Grade Levels

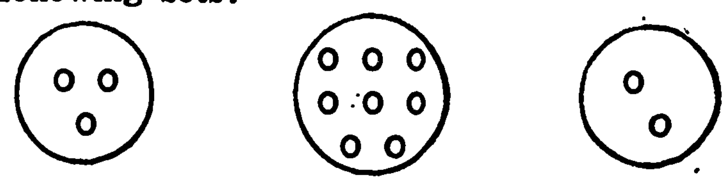
	K	1	2	3	4	5	6	7	8
Items	1 to 4	1 to 13	6 to 17	13 to 22	15 to 27	19 to 33	25 to 41	29 to 44	32 to 44

Objectives

1. Associate numerals with appropriate sets. (After child associates spoken number names with appropriate sets.)
2. Read numerals on clock.
3. Identify the symbols 0 through 9 when the name is given orally.
4. Read and write numerals to 10.
5. Match numerals with sets of tens and ones up to 99.
6. Illustrate the tens and ones represented by any numeral up to 99.
7. Read and write numerals for sets of 10 up to 99.
8. Tell the meaning of each digit for numerals representing numbers less than 100.
9. Give different names for numbers up to 99.
10. Count by 2's and 5's to 30.

Examples

1. Child chooses a card containing the numeral "5" and places it beside the set containing five toys.
2. It is story time when the hour hand points at 9.
3. Teacher says, "Point to the numeral that means '5'."
4. Write the numeral for the number of the following sets:



_____ _____ _____

5. Match the following:

//////////			25
//////////	////		10
//////////	//////////	/////	14

6. Draw a picture which shows bundles of tens and ones for 26, 31, . . . etc.

////////// ////////// //////

7. Name the numerals for the following:

//////////	//////////	///	23
//////////	///////		17

8. The children bundle sixty-two popsicle sticks into six bundles of ten with two left over and read the numeral which shows how many tens and how many ones. "62" is "sixty two" meaning "six tens and two ones."
9. Give different names for 13.
(5 + 8, 15 − 2, 10 + 3, etc.)
(Note: 10 + 3 also makes use of sets of tens and ones.)
10. Start with 4 and count by 2's to 20.
Start with 0 and count by 5's to 30.

Numeration: Performance Objectives (Continued)

Objectives	Examples
11. Count by ones or tens up to 100. (or more)	11. Start with 79 and count by ones to 100. Start with 30 and count by tens to 100.
12. Count in sequence past 100.	12. 98, 99, 100, ____, ____, ____. 406, 407, 408, ____, ____, ____.
13. Count by 2's, 5's, 10's up to 100 or more.	13. 84, 86, 88, ____, ____, ____. 70, 75, 80, ____, ____, ____. 60, 70, 80, ____, ____, ____.
14. Read and write any numeral up to 999.	14. "Write the numerals as I dictate them—74, 809, . . . Read the numerals I have written on the chalkboard."
15. Tell the meaning of any digit in a 3-place numeral.	15. In 234, the 3 represents 3×10 and the 4 represents 4×1.
16. Tell how many hundreds, tens, and ones are represented by any numeral up to 999.	16. 596 means 5 hundreds, 9 tens, and 6 ones, or $596 = 500 + 90 + 6$.
17. Compare Hindu-Arabic numerals with Roman numerals. (I to XXX)	17. 1, I; 2, II; . . . 10, X; etc. Write the corresponding Roman numerals and Hindu-Arabic numerals for numbers 1 through 30.
18. Write expanded notation for any 2- or 3-place numeral with words as place-value symbols; also write as numerals.	18. 282 = 2 hundreds, 3 tens, 2 ones. $282 = 200 + 30 + 2$.
19. Use expanded notation to show regrouping.	19. $282 = 200 + 70 + 12$. 282 = 28 tens and 2 ones.
20. Express orally an understanding of place value to 99,999. (or more)	20. 99,999 means 9 ones, 9 tens, 9 hundreds, 9 thousands, 9 ten-thousands.
21. Use the dollar and cents notation in addition and subtraction problems.	21. John got $10.00 for his birthday. He spent $7.86 at the toy store. How much change did he get from the ten dollars he gave the clerk?
22. Write the corresponding Roman and Hindu-Arabic numerals for numbers 1 to 150, or more.	22. XL ______, XC ______, 149 ______
23. Express orally an understanding of place value to 999,999,999.	23. From charts or other devices the child may say, "The '9' means nine ones," . . . etc.

$$\begin{array}{r}
9\\
90\\
900\\
9000\\
90000\\
900000\\
9000000\\
90000000\\
\underline{900000000}\\
999{,}999{,}999
\end{array}$$

Numeration: Performance Objectives (Continued)

Objectives	Examples
24. Use expanded notation to show regrouping of tens, hundreds, thousands . . . etc.	24. $3{,}423 = 3{,}000 + 400 + \square + 3$.
25. Compare place values and decide the relation symbol which should be placed between pairs of numerals. (hundreds, thousands, ten thousands)	25. $60{,}723 < 67{,}146$ $79{,}901 > 79{,}899$
26. Write the Roman numeral which corresponds to any given Hindu-Arabic numeral for numbers less than or equal to 3000.	26. Express as Roman numerals: 34 __________ 94 __________ 489 __________
27. Write the Hindu-Arabic numeral which corresponds to a given Roman numeral.	27. Express as Hindu-Arabic numerals: LXXVII ________ CMICIX ________
28. Indicate the digit, place value of the digit, and the number which the digit represents in any numeral for numbers from hundredths to millions.	28. In the numeral 1,345,457.76, the 1 is in millions place and represents 1,000,000 and the 6 is in hundredths place and represents .06.
29. Write the expanded numeral for numbers from hundred to millions.	29. Write the expanded numeral for 3,567. Acceptable answers are either: $3(1000) + 5(100) + 6(10) + 7(1)$ or $3(10 \times 10 \times 10) + 5(10 \times 10) + 6(10) + 7(1)$ or $3(10)^3 + 5(10)^2 + 6(10) + 7(1)$.
30. Write the base five numeral for a number whose base ten numeral is given.	30. Write the base five numeral for the set of stars. * * * * * * * * * * * * * $\underline{23}$ five
31. Read Hindu-Arabic numerals from thousandths to billions.	31. Read the following numerals: 1,236,508 1/1000 2/200 2,694,964,467
32. Write Hindu-Arabic numerals from thousandths to billions.	32. Four hundred thousand, twenty-six and six tenths is written 400,026.6.
33. Express the place value of each digit from thousandths to billions.	33. In the numeral 246.135, the digit in the hundreds place is 2, in tens place is 4, etc.
34. Write the expanded notation for any Hindu-Arabic numeral from thousandths to billions.	34. 8,245.53 in expanded notation is $8 \times 1{,}000 + 2 \times 100 + 4 \times 10 + 5 \times 1 + 5 \times \frac{1}{10} + 3 \times \frac{1}{100}$.
35. Express place value of whole numbers as a polynomial in powers of 10 and units.	35. $456 = 4(10^2) + 5(10) + 6$.

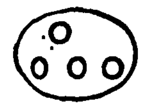
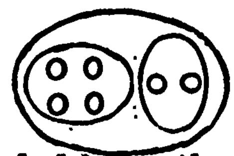
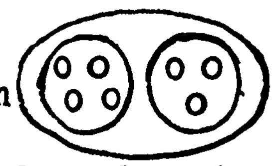
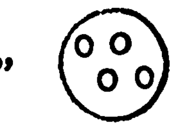
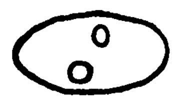
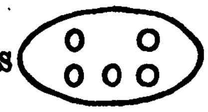
Numeration: Performance Objectives (Continued)

Objectives	Examples
36. Write decimal numbers to thousandths, as a polynomial in powers of tenths.	36. $.7 = 7(\frac{1}{10})$ $.81 = 8(\frac{1}{10}) + 1(\frac{1}{100})$ $.236 = 2(\frac{1}{10}) + 3(\frac{1}{10})^2 + 6(\frac{1}{10})^3$
37. Write some fractions as terminating decimals.	37. 4/5 = .80 3/4 = .75
38. Express numbers in different bases.	38. In this set of tally marks {//////////} there are <u>10</u>, base ten; <u>1010</u>, base two; <u>20</u>, base five.
39. State the place value of each digit in a base ten numeral from billions to ten thousandths.	39. In the numeral 12,345.7689, the 2 is in the thousands place, and the 8 is in the thousandths place.
40. Write any base ten numeral in expanded notation.	40. 234.56 in expanded notation is $2 \times 100 + 3 \times 10 + 4 \times 1 + 5 \times \frac{1}{10} + 6 \times \frac{1}{100}$.
41. Write fractions as either terminating or repeating decimals.	41. $1/8 = .125$, $2/11 = \overline{.18}$, $2/7 = \overline{.285714}$
42. Represent a given positive rational number using decimal, common, mixed-numeral, percent, and scientific notation.	42. 3/5 .6 60% 2.35 235/100 235% 35% 35/100 .35 7/5 1.4 140% 1 2/5 1.4×10^{0}
43. Write a non-repeating decimal numeral.	43. .1010010001 . . .
44. Distinguish between decimal notation for rational numbers and irrational numbers.	44. 2.4313131 . . . represents a rational number. .101003000 . . . represents an irrational number. How do you recognize a rational or irrational number in decimal form?

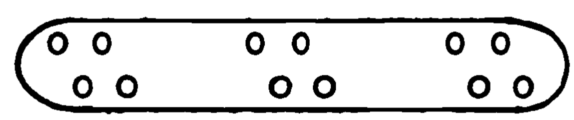
Operations and Properties: Performance Objectives

Suggested Grade Levels

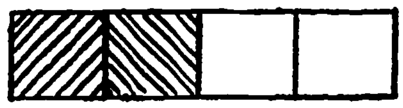
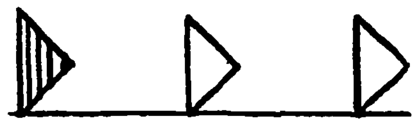
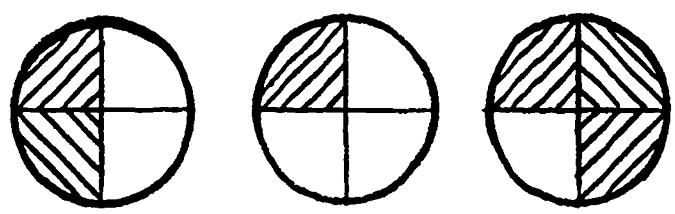
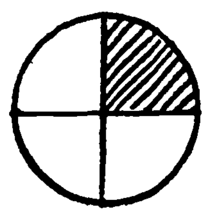
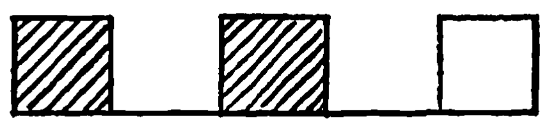
	K	1	2	3	4	5	6	7	8
Items	1 to 3	1 to 8	9 to 14	13 to 23	17 to 33	32 to 45	46 to 50	48 to 53	51 to 57

Objectives	Examples
1. Combine two sets into a single set (introduction to addition).	1. Use bottle caps to form sets that look like  and  Then combine the two sets.  Demonstrate first with physical objects, then draw a picture to illustrate. (The ability to count is not necessary here.)
2. Separate some members (subset) from a set (introduction to subtraction).	2. From  "take away"  and show the set which remains. Demonstrate first with physical objects and then illustrate with a drawing. (The ability to count is not necessary here.)
3. Demonstrate a set which when combined with a given set produces another given set.	3. Produce the set which when combined with  gives  Answer 
4. Use addition facts up to 10.	4. Introduction and basic use of the addition facts using counters or other aids.
5. Identify examples of the commutative property.	5. The order in which the addends are arranged does not affect the sum. Thus, $4+3=3+4$.
6. Use the subtraction facts up to 10.	6. Introduction and use of the basic addition facts using counters or other aids.
7. Show the relationship between addition and subtraction.	7. Given three numbers such as 3, 4, 7, the related number sentences such as $3+4=7$ and $7-4=3$ should be demonstrated.
8. Use the associative property in some examples.	8. Illustrate that $2+(7+3)=(2+7)+3$.
9. Use the addition facts (0 – 18) to add numbers through hundreds without regrouping.	9. Add without regrouping. (a) $\begin{array}{r} 9 \\ +8 \\ \hline \end{array}$ (b) $\begin{array}{r} 342 \\ +546 \\ \hline \end{array}$

Operations and Properties: Performance Objectives (Continued)

Objectives	Examples
10. Use the subtraction facts (0 – 18) to subtract numbers without regrouping.	10. Subtract without regrouping $\begin{array}{r} 645 \\ \underline{-234} \end{array}$
11. Show the relationship between addition and subtraction.	11. Write the two addition sentences using the numbers (8, 7, 15). Write the associated subtraction sentence for each addition sentence.
12. Use the commutative and associative properties in addition.	12. The student should be able to rewrite $4 + 7$ as $7 + 4$; identify $4 + 7 = 7 + 4$ as an example of the commutative (order) property; identify $(2 + 3) + 7 = 2 + (3 + 7)$ as an example of the associative (grouping) property.
13. Recognize a simple multiplicative situation.	13. Jean has 3 sets of dishes. Each set has 4 dishes. Draw a diagram showing how many dishes Jean has. 
14. Identify some multiplicative situations.	14. Given  Find subsets of two elements. Identify the number of subsets of two elements.
15. Recognize and use the commutative (order) and associative (grouping) properties in addition with numbers through hundreds; also, add three or more two-digit numbers in column.	15. (a) $\begin{array}{r} 548 = 500 + 40 + 8 \\ \underline{+374 = 300 + 70 + 4} \\ 800 + 110 + 12 = 900 + 20 + 2 = 922 \end{array}$ (b) $a + b = b + a$ $(a + b) + c = a + (b + c)$
16. Apply zero (0) as the identity element in addition.	16. What number added to 6 will give a sum of 6? Answer: zero (0). Students should be able to write the number sentences $0 + 6 = 6$ and $6 + 0 = 6$.
17. Regroup in expanded notation to subtract numbers through hundreds. Also express the missing addend (difference) in the shorter form.	17. $\begin{array}{rl} 640 = & 600 + 40 + 0 = 600 + 30 + 10 \\ \underline{-532} = & \underline{500} + \underline{30} + \underline{2} = \underline{500} + \underline{30} + \underline{2} \\ 108 = & \qquad\qquad\quad\; 100 + 0 + 8 \end{array}$
18. Check addition and subtraction examples using the relationship between addition and subtraction.	18. $640 - 532 = 108$ because $108 + 532 = 640$ $234 + 197 = 431$ because $431 - 197 = 234$.
19. Find products using (a) Factors 0 – 5. (b) commutative property. (c) associative property with products that are multiples of 10 or 100 (d) distributive (multiplication over addition) property with factors 0 – 5 and multiples of 10.	19. (a) $4 \times 3 = 12$ (b) $4 \times 5 = 5 \times 4$ (c) $\begin{aligned} 3 \times 50 &= 3 \times (5 \times 10) \\ &= (3 \times 5) \times 10 \\ &= 15 \times 10 \\ &= 150 \end{aligned}$ (d) $\begin{aligned} 4 \times 13 &= (4 \times 10) + (4 \times 3) \\ &= 40 + 12 \\ &= 52 \end{aligned}$

Operations and Properties: Performance Objectives (Continued)

Objectives	Examples
20. Use division facts for factors 0 – 6.	20. 18 divided by 3 = 6
21. Show the relationship between multiplication and division.	21. Write the multiplication sentences using 2, 5, 10. Write the associated division sentence for each multiplication sentence.
22. Use "patterns" to distribute two or three-digit dividends to divide.	22. $48 \text{ divided by } 4 = (40 \div 4) + (8 \div 4) = 10 + 2 = 12$
23. Extend the division algorithm to vertical form. Use division as repeated subtraction and remainders of 0.	23. $\begin{array}{r l} 12 & \text{R0} \\ 4\overline{)48} & \\ \underline{40} & \underline{10} \\ 8 & \\ \underline{8} & \underline{2} \\ 0 & \end{array}$
24. Use the commutative and associative properties of addition with two, three, or four-digit addends.	24. $364 + 123 = 123 + 364$ $(a + b) + c = a\ (b + c)$
25. Choose the fraction which correctly identifies that part of the figure which is shaded.	25. (a) Choose the fraction which represents the shaded part of the figure.  $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ (b) Choose the fraction which represents the shaded part of the figure.  $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
26. Choose the picture where that part of the figure which is shaded denotes a given fraction.	26. Choose the diagram where the shaded part represents 
27. Write the fraction which expresses that part of the figure which is shaded.	27. (a)  (b) 

Operations and Properties: Performance Objectives (Continued)

Objectives / **Examples**

Objective 28. Use given figures to show that two given fractions are equivalent.

Example 28. Use the figures below.

NOTE to teachers: The figures used must be congruent.

Figure 1 has two congruent regions. Figure 2 has four congruent regions. Shade one of the regions in Figure 1. Write the related fraction. Shade enough of Figure 2 so that the shaded regions match. Write the related fraction. How are the fractions related (equivalent)?

Objective 29. Apply the commutative property for multiplication and the generalizations concerning the multiplicative properties of 0 and 1 with the mastery of the multiplication facts.

Example 29.

$a \times b = b \times a$

$0 \times n = 0$

$1 \times 5 = 5$

Objective 30. Divide selected numbers by using the related multiplication facts.

Example 30.

(a) $7 \times 8 = 56 \quad 56 \div 7 = 8$

(b) $60 \times 7 = 420 \quad 420 \div 7 = 60$

Objective 31. Apply the distributive law to find the product of a two- or three-digit factor and a one-digit factor.

Example 31.

$$\begin{aligned} 9 \times 81 &= (9 \times 80) + (9 \times 1) \\ &= 720 + 9 \\ &= 729 \end{aligned}$$

Objective 32. Using numbers through thousands, use the traditional multiplication algorithm.

Example 32.

```
 268
 ×49
```

Objective 33. Divide using repeated subtraction and one- or two-digit divisors.

Example 33.

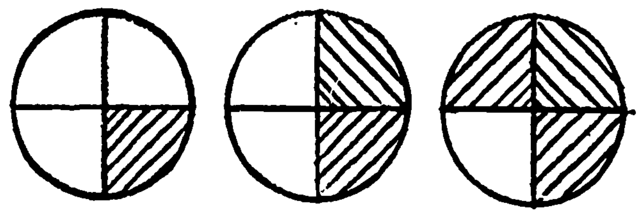
```
       42  R3
23 | 969
     920  40
     ---
      49
      46   2
     ---
       3
```

Objective 34. Use the traditional algorithm to divide.

Example 34. Divide 1987 by 42.

Objective 35. Write and order the fractions related to given diagrams.

Example 35. Write the fractions related to each of the following.



Order the fractions.

Objectives	Examples
36. Find the least common denominator (LCD) of several fractions by using the least common multiple (LCM) of the denominators.	36. To find the LCD for $\frac{1}{12} + \frac{1}{10} + \frac{3}{35}$, first find the LCM of 12, 10, and 35. First find the prime factorization. $12 = 2 \times 2 \times 3$ $10 = 2 \times 5$ $35 = 5 \times 7$ The LCM consists of every different prime factor in the above factorizations and each factor is in the LCM the least number of times it is found by any *one* of the prime factorizations. Thus, the LCM of 12, 10 and 35 is $2 \times 2 \times 3 \times 5 \times 7$ or 420.
37. Arrange several fractions in order from smallest to largest and vice versa.	37. (a) Arrange $\frac{2}{3}, \frac{7}{3}, \frac{5}{3}, \frac{2}{3}$ so the numbers represented go from the smallest to largest. (b) Which of $\frac{2}{3}$ and $\frac{1}{2}$ represents the smaller number?
38. Perform addition, subtraction, multiplication and division with fractions.	38. (a) Find the sum of $\frac{3}{8}$ and $\frac{2}{8}$. (b) Find the difference of $\frac{1}{2}$ and $\frac{1}{3}$.
39. Identify addend, sum, factor, product.	39. In $2 \times 3 = 6$ the 2 is called a(n)__________. The 6 is called the___________________.
40. Perform multiplication by the multiples of a basic number such as 20, 300, etc., by using the basic multiplication facts, the associative and commutative properties, and multiplication by powers of 10.	40. $3 \times 20 = 3 \times (2 \times 10) = (3 \times 2) \times 10 = 6 \times 10 = 60$
41. Estimate products and quotients.	41. The product of 23 and 6 is > 120 but < 180, or is between 120 and 180.
42. Identify and use the commutative, associative, and distributive properties and the properties of 1 and 0.	42. State the principle illustrated in each of the following. $23 + 457 = 457 + 23$ $43 \times (4 \times 1) = 43 \times 4$
43. Write the decimal fraction equivalent to a fraction and vice versa.	43. Write the decimal form of $\frac{3}{4}$.
44. Arrange several decimal fractions in order.	44. Arrange 0.3, 0.03, 0.029 so that the numbers represented go from smallest to largest.
45. Perform the four fundamental operations with decimal fractions.	45. Complete to make true sentences. $3.4 + 5.4 = \boxed{}$ $42 + 6.1 = \boxed{}$

Objectives	Examples
46. Estimate and perform additions, subtractions, multiplications, and divisions on the set of non-negative rational numbers.	46. $\frac{3}{4}+\frac{5}{6}+\frac{2}{3}=\frac{27}{12}$ $4\frac{1}{3}-2\frac{5}{6}=1\frac{1}{2}$ $\frac{4}{5}\times\frac{8}{7}=\frac{32}{35}$ $\frac{5}{8}\div\frac{2}{3}=\frac{15}{16}$
47. Compute the sum, difference, product and quotient using decimals.	47. (a) $0.247+0.132=\boxed{}$ (b) $0.946-0.759=\boxed{}$ (c) $0.6\times 0.21=\boxed{}$ (d) 5.26 divided by 0.18 = $\boxed{}$ (d) $5.26\div 0.18=\boxed{}$
48. Order a set of rational numbers.	48. Arrange 0.73, , 0.075, 7.5 so that the numbers which are represented are in order from smallest to largest.
49. Compute the sum, product, difference, and quotient of any two whole numbers.	49. (a) $25{,}665+39{,}672=\boxed{}$ (b) $325\times 256=\boxed{}$ (c) $3{,}246-1{,}738=\boxed{}$ (d) $2{,}170\div 62=\boxed{}$
50. Multiply mentally by powers of ten.	50. $100(23.5)=2350$ $.01(3.4)=.034$
51. Compute the sums, products, differences, and quotients of non-negative rational numbers in fractional form where the numerator and denominator are less than 50.	51. $\frac{3}{8}+\frac{5}{7}=\frac{61}{56}$ $\quad\frac{2}{3}\times\frac{7}{3}=\frac{14}{9}$
52. Compute the sum and product of integers between -20 and 20. NOTE to teachers: Extend the whole numbers to the set of integers by using the definition of the additive inverse and by making use of patterns and the number line. DO NOT give just rules of operations.	52. $8+(-5)=3$ $(-7)+3=-4$ $(-7)(-7)=49$ $(8)(-6)=-48$
53. Compute the sum, product, difference, and quotient of any two non-negative rational number in decimal notation. Note: a non-negative rational number is 0 or a positive rational number.	53. $3.25+2.8=6.05$ $(.2)(.3)=.06$ $8.23-5.42=2.81$ $.45\div 1.5=.3$
54. Compute the sum, product, difference, and quotient of any two non-negative rational numbers in decimal notation.	54. $2.35+15.2=\boxed{}$ $(.25)(.04)=\boxed{}$ $18.3\div 2.54=\boxed{}$

Operations and Properties: Performance Objectives (Continued)

Objectives	Examples
55. Compute the sum, product, difference, and quotient of any two non-negative rational numbers in fractional form with denominators less than one hundred.	55. $\frac{2}{75}+\frac{3}{45} = \square$ $(\frac{3}{17}) \cdot (\frac{37}{30}) = \square$
56. Compute the sum, product, difference, and quotient of a pair of integers between –100 and 100.	56. $85 + (-15) = \square$ $10 \times (-10) = \square$ $(-80) - (-30) = \square$ $(-80) - (-5) = \square$
57. Identify and illustrate the commutative, associative and distributive properties for rational numbers. Also the multiplicative and additive properties of 0 and 1. Additive and multiplicative inverses.	57. $\frac{5}{8} + \frac{11}{6} = \frac{11}{6} + \frac{5}{8}$ $\frac{2}{5}(\frac{3}{8}+\frac{1}{7}) = \frac{2}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{1}{7}$

Mathematical Sentences: Performance Objectives

Suggested Grade Levels

	K	1	2	3	4	5	6	7	8
Items	1 to 3	1 to 6	5 to 11	8 to 15	12 to 18	15 to 22	18 to 25	26 to 30	26 to 30

Objectives

1. Use and interpret mathematical symbols of >, <, =, + and −.

2. Make a diagram that illustrates a given number sentence; also, given the diagram, write the corresponding number sentence.

3. Illustrate word problems by a drawing; then translate into an open sentence and solve.

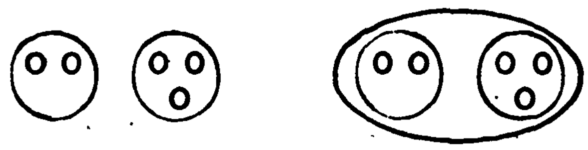
4. Solve open sentences using the addition facts up to 10.

5. Find several entries so that $\triangle + \square = 8$ becomes a true number sentence.

Examples

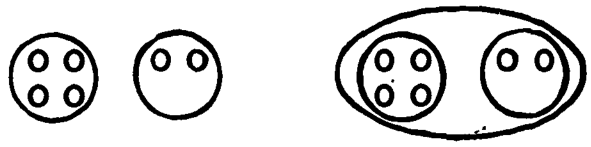
1. Use the symbols >, <, =, + and − to make the following true sentences.
   (a) $3 \square 5$
   (b) $3 \square 2+1$
   (c) $4=5 \square 1$
   (d) $3+2+5 \square 1$

2. Draw a picture to illustrate the number sentence $2+3=5$.



Note to teachers: Distinguish between a number sentence and an open sentence. For example, $2+1=3$ is a number sentence while $\square+1=3$ is an open sentence.

3. David had 4 marbles and found 2 more. How many marbles does David have now?



$4+2=\square$

4. Find the solutions for:
   $2+\square=7$ $\square-4=3$
   $8-3=\square$ $\square+2=8$
   Note to teachers: A simple approach might be to relate the open sentence to a true number sentence, e.g., $2+\square=7$ can be related to $2+5=7$, a true number sentence. Therefore, the box and 5 can represent the same number.

5. If $\triangle + \square = 8$ is a true number sentence, the left side wil be one of the following: 0 + 8, 1 + 7, 2 + 6, 3 + 5, 4 + 4, 5 + 3, 6 + 2, 7 + 1, or 8 + 0.

Mathematical Sentences: Performance Objectives (Continued)

Objectives	Examples
6. Give a word problem for a given number sentence.	6. A word problem for $4 + \square = 7$ might be: Mary had 4 sea shells. She found some more on the beach. When she counted them she had 7. How many did she find?
7. Solve open sentences using addition facts up to 18.	7. Find solutions for: $4 + 13 = \square$, $\square + 7 = 28$, $13 - \square = 7$. Include open sentences of the type $\square + 1 = 1 + \square$ where every number satisfies.
8. Write subtraction sentences which result from a given addition statement and vice versa.	8. Given $3 + 8 = \square$, then $\square - 8 = 3$ and $\square - 3 = 8$. Given $\square - 3 = 5$, then $3 + 5 = \square$ and $5 + 3 = \square$.
9. Determine whether a mathematical sentence is true or false.	9. Mark T for true and F for false after the following sentences. $32 + 48 = 175 - 95$ $87 + 46 = 77 + 77$.
10. Complete mathematical sentences so that they are true.	10. Use a <, >, or = to make the following sentences true. $43 + 86 \square 79 + 68$ $256 + 767 \square 413$.
11. Solve and interpret the solution of a mathematical sentence.	11. $178 - 57 = \square$ $178 - 57 = 121$ Michele's father is 121 pounds heavier than she is.
12. Write division sentences which result from a given multiplicative statement and vice versa.	12. Given $7 \times 4 = \square$, then $\square \div 7 = 4$ and $\square \div 4 = 7$. Given $20 \div 4 = \square$, then $\square \times 4 = 20$, etc.
13. Write open sentences which describe a given array.	13. Given $\begin{array}{cccc} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{array}$ then $\begin{array}{r} 4 \times 5 = \square \\ \square \div 4 = 5 \\ \square \div 5 = 4 \\ 5 \times 4 = \square \end{array}$ Note to teachers: A simple approach might be to relate the open sentence to a true number sentences, e.g., $4 \times \square = 20$ can be related to $4 \times 5 = 20$. Therefore, the box and 5 can represent the same number.
14. Write open sentences expressing the basic operation for story problems.	14. Andy played 16 records for a party. Each one lasted 18 minutes. How many minutes of music were on these records? $16 \times 18 = \square$.

Mathematical Sentences: Performance Objectives (Continued)

Objectives	Examples
15. Express different true statements from one story problem.	15. If Mark is 52 inches tall and Bob is 63 inches tall, how many inches taller is Bob than Mark? $63 - 52 = 11$, $11 + 52 = 63$ $63 > 52$ $11 < 63$ Bob is taller. Mark is shorter.
16. Express true sentences using multiplication factors in multiples of tens or hundreds.	16. $7 \times 90 < 647$
17. Determine the correct relation symbol ($<$, $>$, $=$, $\neq$) for some pairs of whole numbers and fractions.	17. $60 \times 6 < 372$ $1{,}640 > 1{,}539$ $\frac{3}{6} = \frac{1}{2}$ $\frac{2}{3} > \frac{1}{3}$
18. Write numerals for place holders in every possible position to make true sentences and indicate inverse operations.	18. $\square + 36 = 84$ $360 - 4 = \square$ $84 - 36 = \square$ $\square \times 4 = 360$
19. Determine the truth or falsity of mathematical sentences using $<$, $>$, $=$, $\neq$, $\nless$, $\ngtr$, whole numbers, fractions, decimals, and the four fundamental operations.	19. State whether each of the following is true or false and write a true sentence if the given one is false. $43 \times 6 > 240$ $265 \times 4 = 1040$
20. Write open sentences expressing the relationship(s) in stated problems.	20. Write the open sentence which could be used to solve: There are 1008 seats in an auditorium. The seats are arranged in rows with 21 seats in each row. How many rows are there?
21. Write equivalent mathematical sentences using inverse operations.	21. Write an equivalent number sentence using subtraction for $276 + 25 = 301$. Write an equivalent open sentence for $6 \times \square = 42$.
22. Determine the solution set for selected mathematical sentences. (Solution sets may be sets of one member, infinite sets, or the empty set.)	22. The solution set for $543 - \square = 543$ is the set consisting of the number 0. The solution set for $\square - 10 = 4$ is the empty set if negative numbers are not used. The solution set for $\square \times 3 = 3 \times \square$ consists of all the numbers they know.
23. Solve simple linear equations.	23. $2n + 5 = 19, n = 7$ $2(n + 5) = 18, n = 4$.
24. Apply the concept of percent in solving various equations.	24. 45% of $6 = n$, 15% of $n = 90$, $n\%$ of $4500 = 135$.
25. Write a mathematical sentence which could be used to solve a verbal problem.	25. The sum of two numbers is ten. The first number plus twice the second number is sixteen. Find the two numbers.

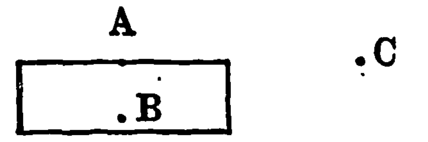
Mathematical Sentences: Performance Objectives (Continued)

Objectives	Examples
26. Solve simple linear equations.	26. $n + 3 = 8$, $2n + 5 = 13$.
27. Write a mathematical sentence which could be used to solve a verbal problem.	27. A 12-ft. board is cut into three pieces. Two of the pieces are the same length and the third is three feet longer than each of the other two. How long is each piece? $n + n + (n + 3) = 12 \qquad n = 3$
28. Use the correct symbol, =, <, >, to form true equations and inequalities.	28. $4 + 5 \quad 6 + 3 \qquad 12 + 4 \quad 3 \times 2$ $16 - 12 \quad 3 \times 1$
29. Write one mathematical sentence which could be used to solve a verbal problem involving two or more steps.	29. John bought a tablet for 59¢ and six erasers costing 10¢ each. What was his bill? $59 + (6 \times 10) = n$, $n = 119¢ = \$1.19$.
30. Correctly interpret and label the solution of a mathematical sentence in problem solving.	30. Sue is 56 inches tall and Kate is 61 inches tall. How much taller is Kate than Sue? $61 - n = 56$, $n = 5$; 5 inches taller.

Geometry: Performance Objectives

Suggested Grade Levels

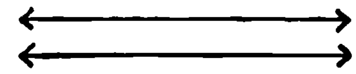
	K	1	2	3	4	5	6	7	8
Items	1 to 2	1 to 6	5 to 11	7 to 18	13 to 24	20 to 31	25 to 35	35 to 49	35 to 49

Objectives	Examples
1. Recognize models of geometric figures and describe some of their properties.	1. Child finds objects in his environment which represent geometric figures: Hoop, bulletin board frame, edge of floor tile, "triangle" musical instrument, shoe box, blocks, ball, etc.
2. Identify the circle, square, rectangle, and triangle.	2. After identifying and naming various geometric shapes, the pupil could draw the various figures.
3. Recognize betweenness relations on a line.	3. Use tinkertoys to illustrate that, of three points on a line, only one is between the other two.
4. Tell when a curve is a simple closed curve or an open curve.	4.  Simple closed curve  Open curve
5. Describe a segment, line, ray, and an angle.	5. A segment has two endpoints while a line goes on infinitely in two directions.
6. Recognize some of the properties of geometric figures.	6. While both a square and a rectangle have four corners, they are different. Why?
7. Distinguish between geometric abstractions and models of them.	7. A stretched string is not a segment but is a model of a segment.
8. Identify the circle, triangle, square, rectangle, and other simple polygons, recognizing some of their properties.	8. Tell how a circle differs from a square. Tell what is the same about a rectangle and square.
9. Recognize that two different points determine a unique line and that two different lines may intersect in exactly one point.	9. •A •B How many lines can contain both point A and point B?
10. Distinguish between a simple closed curve and non-simple or non-closed curve, and recognize the plane separation properties of simple closed curves and polygons.	10.  Which point is in the interior of the rectangle? the exterior? neither?

Objectives	Examples
11. Recognize betweenness relations on a line.	11. A segment is made up of two points and all the points between them.

12. Objectives: Recognize parallel and intersecting lines.

12. Examples:

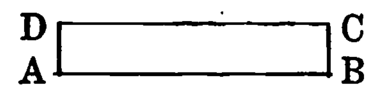


Two lines in the same plane which never intersect are said to be parallel.

Objectives	Examples
13. Identify and classify angles in relation to right angles.	13. An angle which measures less than a right angle is called an acute angle.

14. Objectives: Identify and name polygons by the number of their sides.

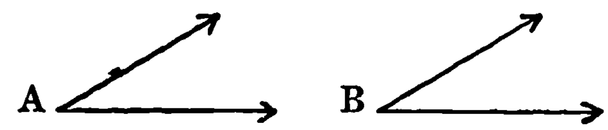
14. Examples: This polygon contains how many sides?



It is called a____________________.
Does this name apply to all polygons having four sides?

15. Objectives: Determine the congruence of segments by comparing lengths, and the congruence of angles by superposition.

15. Examples:

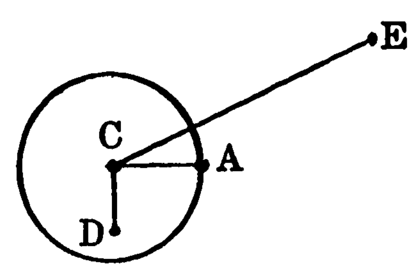


See if ∠ A is congruent to ∠ B by tracing ∠ B on another piece of paper and placing it over ∠ A.

Objectives	Examples
16. Describe the similarities and differences among parallelograms, rectangles, and squares.	16. How does a square differ from an ordinary rectangle?

17. Objectives: Define a circle and illustrate its parts.

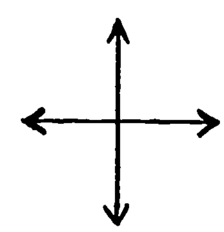
17. Examples:



$\overline{AC}$ is a radius
$\overline{CD}$ and $\overline{CE}$ are not radii

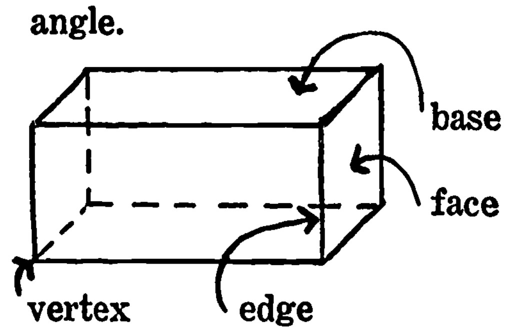
18. Objectives: Define parallel and perpendicular lines using intuitive concepts.

18. Examples: These two lines are perpendicular because they form a right angle.

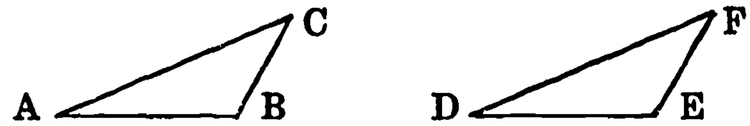


19. Objectives: Describe and name the parts of the sphere and rectangular prism.

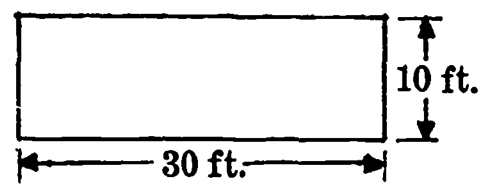
19. Examples:



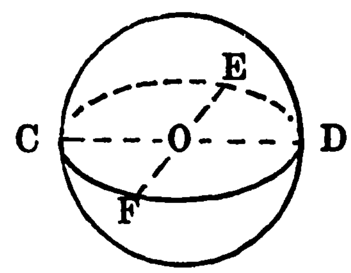
Objectives	Examples
20. Classify triangles and quardrilaterals according to their angles and sides, and distinguish some of their properties.	20. a. What is an acute triangle? b. Does a parallelogram differ from a square? How?
21. Determine the congruence of two triangles by superposition or measurement.	21. These two triangles would fit exactly if one were cut out and placed over the other. What is the relation between corresponding pairs of sides? of angles?



Objectives	Examples
22. Find the perimeter of a polygon and the area of a rectangle by counting unit squares.	22. If you had a garden like the one shown, how much edging would you have to buy for it? How many square feet are in your garden?



Objectives	Examples
23. Describe and name the parts of the sphere, rectangular prism, cone and cylinder.	23.

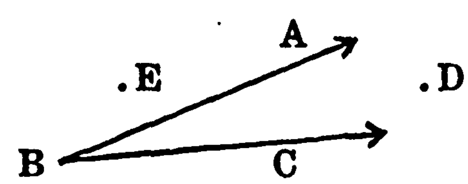


$\overline{CD}$ is a________

$\overline{EF}$ is a________

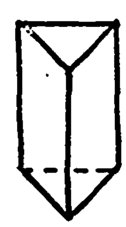
$\overline{OD}$ is a________

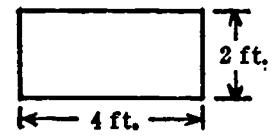
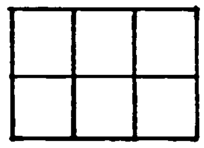
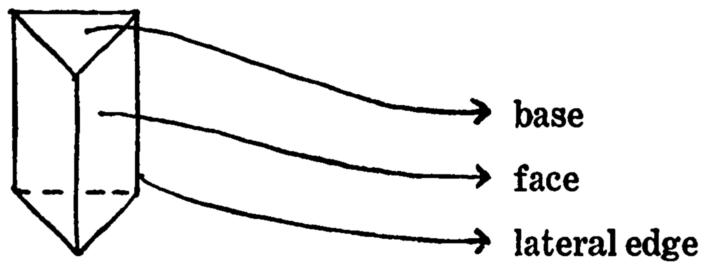
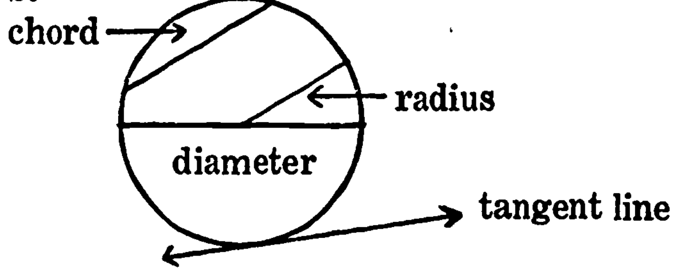
Objectives	Examples
24. Identify segment, ray, line, plane, and space; and state their various interrelationships.	24. If a line intersects a plane not containing it, the intersection is a single point.
25. Identify and classify angles by their measures.	25. a. An angle is the figure formed by two rays with a common endpoint. b. An obtuse angle is one whose degree measure is greater than 90.
26. Recognize the plane separation properties of angles, polygons, and simple closed curves, and the separation of space by simple closed surfaces.	26.



Which points are in the interior of $\angle ABC$?

Objectives	Examples
27. Classify triangles and quadrilaterals according to their special properties.	27. a. Draw an isosceles right triangle. b. Under what circumstances can a parallelogram be a square?
28. Recognize and name solid figures such as right prism and right circular cone.	28. This figure is a triangular prism because________ ________________.



Objective	Examples
29. Compute the perimeters of squares, rectangles, and triangles.	29. Can you find a quick way to find the perimeter of this rectangle? 
30. Compute the areas of squares, rectangles, and triangles by counting unit squares, and the volume of a rectangular prism by counting unit cubes.	30. Since there are 6 unit squares in this rectangle, we say that its area is 6 square __________. 
31. Define and use intuitively parallel and perpendicular lines and planes.	31. When is a line parallel to a plane?
32. Name the parts and state the properties of right prisms, cylinders, pyramids, and cones.	32.  The faces of a prism are____________. The bases of a cylinder are____________.
33. Compute: a. perimeters of plane figures by measurement and by formula; b. areas of plane figures by counting unit squares and by formula; c. volumes of space figures by counting unit cubes and by formula.	33. What unit would be used to: a. find perimeters of parallelograms and regular polygons? b. find areas of rectangles? c. find volumes of rectangular prisms?
34. Compute the circumference of a circle.	34. Cut a circle out of cardboard. Find its circumference by finding the length of a piece of string that will just "fit" around the circle, and then measure the diameter of the circle. Divide the circumference by the diameter. Repeat this procedure several times and compare the results. (Note: These results will be approximately the same.)
35. Identify the distinguishing properties of segments, rays, lines, angles, polygons, circles, and spheres.	35. a. Explain the differences between the figures designated by AB, AB, and AB. b. 
36. Identify the interrelationships of points, lines, planes, and other figures in space.	36. a. Two different lines may intersect in a single point. b. Three non-collinear points determine one and only one plane.

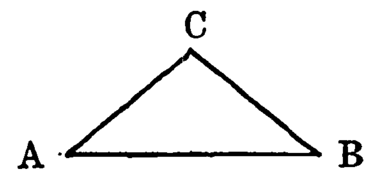
Objective	Examples

37. State the distinction between congruence and equality of geometric figures with emphasis on segments, angles, and triangles.

37. If $\overline{AB} = \overline{CD}$, is $\overline{AB} \cong \overline{CD}$?
If $\overline{AB} \cong \overline{CD}$, is $\overline{AB} = \overline{CD}$?

38. State the sufficient conditions for congruence of triangles.

38. Construct a triangle whose sides are congruent to the sides of this triangle. Is your triangle congruent to this one?

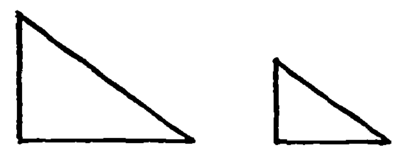


39. Use compass and straight-edge to perform the standard Euclidean constructions.

39. a. A segment congruent to a given segment.
b. An angle congruent to a given angle.
c. Etc.

40. Use similar triangles intuitively.

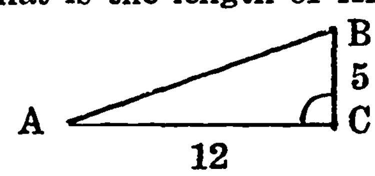
40.



These two triangles are not congruent because they are different in size. They are ______________________.

41. State and apply the Pythagorean Theorem.

41. What is the length of $\overline{AB}$?

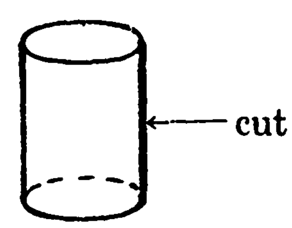


42. Name the parts and state the properties of cylinders, prisms, cones, pyramids, and polyhedra.

42. a. A dodecahedron has how many faces?
b. Are the bases of a prism congruent?

43. Find perimeters and areas of plane figures and surface areas and volumes of prisms and cylinders by formula.

43. If you imagine this cylinder cut open and spread out into a rectangle, how long is its base?

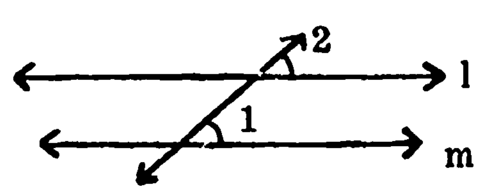


44. Compute the circumference and area of a circle.

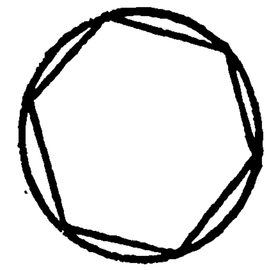
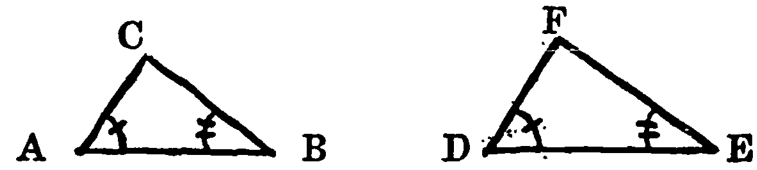
44. What is the area of a circle whose diameter measures 14 in.?

45. Use transversals to two lines and their related angles to establish parallelism.

45.



If $\angle 1 \cong \angle 2$ what can you say about lines l and m?

Objectives	Examples
46. Derive the formulas for the circumference and area of a circle by informal limiting process, and use the formulas for actual computation.	46. If you increase the number of sides in this polygon, can you see that its perimeter will approach the circumference of the circle? 
47. Prove triangles congruent using the SAS, ASA, and SSS conditions.	47.  $\triangle ABC \cong \triangle DEF$ because______________.
48. Use compass and straight-edge to perform the standard Euclidean constructions.	48. a. Copy segments, angles, and triangles. b. Bisect angles and segments. c. Construct perpendiculars. d. Construct parallels.
49. Recognize the sufficient conditions for similarity of triangles, and use their properties to solve practical problems.	49. If a yardstick casts a shadow 4 ft. long at the same time a flagpole's shadow is 24 ft. long, how tall is the flagpole?

Measurement: Performance Objectives

Suggested Grade Levels

	K	1	2	3	4	5	6	7	8
Items	1 to 5	1 to 13	10 to 21	14 to 26	21 to 30	25 to 34	30 to 40	36 to 49	36 to 49

Objectives†	Examples
1. Recognize comparison of size, weight, and temperature of objects.	1. a. What kind of sound does the <u>shortest</u> string on the autoharp make? b. A man is almost as tall as a door. c. John weighs ________ pounds. d. On our balance scales, does the box weigh more than the doll? e. On a snowy morning, does the red mercury in the thermometer go up or down when we bring it inside?
2. Recognize place relationships, speed comparisons.	2. a. Hold the plate over the sink. b. Jill is sitting between Pat and Sue. c. Which is the fastest — hopping, running, or walking? d. Which is slower, a car or a plane?
3. Compare value of coins.	3. How many pennies would buy as much as a nickel?
4. Make simple linear measurements with inches, feet, and yards.	4. a. Compare heights with the length of a yardstick and a foot ruler. b. Compare inch-long strips of paper with lengths of objects.
5. Measure time and name a. Seasons, month and day of month, and day of week.	5. a. Large calendar with removable month names and numerals is brought up to date each day, and future plans are made with its help.
b. Hour and minute measure.	b. It is about time for lunch. Where is the small hand on the clock when we come to school? When we leave to go home? Egg or ove timer can be used to mark time intervals.
6. Tell why we use a standard unit in measurement.	6. Development of the standard after using various units in the room or using children's hands.

† Note: See Geometry Strand for topics about measurements of geometric figures.

Measurement: Performance Objectives (Continued)

Objectives	Examples
7. Compare the values of a penny, a nickel, and a dime.	7. How many suckers can I buy for a penny? For a nickel? For a dime?
8. Measure length to the nearest inch.	8. Using rulers with half-inch markings measure a book, the height of a child, etc.
9. Tell time by the hours.	9. What time do we have lunch? What time do we come to school? Go home?
10. Use the degree as a unit of measure for temperature.	10. When the temperature gets warmer, does the marker in the thermometer go up or down? How cold must it be for water to freeze?
11. Use the pound as the unit of measure for weight.	11. About how many pounds do you weigh?
12. Compare units of liquid measure such as a cup, a pint, and a quart.	12. Using liquid or sand, have the child demonstrate the relationship between various units of measure.
13. Compare inches and feet.	13. How many inches in one foot?
14. Recognize the need for standard units of measure.	14. Use standard measures after experimenting with pupil-made measure.
15. Compare the values of U.S. coins to 50¢.	15. Count money and make change to 50¢.
16. Use and compare common length measures such as inch, foot, and yard.	16. How many inches in a foot? How many feet in a yard?
17. Measure length to the nearest inch.	17. Use rulers marked to the nearest half-inch to measure objects.
18. Tell time to the nearest five minutes, and know the simplest elements of the calendar.	18. How many months until Christmas?
19. Read a temperature scale.	19. Use a thermometer marked in degrees to find the temperature.
20. Use the pound as a unit of weight.	20. Use scales to measure weights to the nearest pound.
21. Compare common measures of capacity such as pint and quart.	21. Use liquid or sand to show relationships between various measures.
22. Use U.S. currency.	22. Use lunch and milk money problems to make change for a dollar.
23. Apply the common measures of length, capacity, weight, money, and time.	23. I weigh 53 pounds. This book is $7\frac{1}{2}$ inches long. There are 12 months in one year. It is 3:13 in the afternoon.

Measurement: Performance Objectives (Continued)

Objectives	Examples
24. Measure length to the nearest half-inch.	24. Use rulers marked to the nearest fourth-inch to measure objects.
25. Show relationships among the units of time and apply the common measures of time to the nearest minute. Use the calendar.	25. There are 6 months in one-half year. How many minutes are there in one hour?
26. Convert commonly used measures to equivalent measures.	26. 1 yard = 36 inches 16 ounces = 1 pound
27. Apply the common English measures of length, capacity (dry and liquid), weight, time, money, temperature, and counting.	27. a. How far is it to the nearest city? b. List the number of days in each month. c. How many days in a leap year? a regular year? d. Make change.
28. Measure lengths to the nearest $\frac{1}{4}$ inch and to the nearest centimeter.	28. Use rulers marked to the nearest eighth-inch and to the nearest millimeter.
29. Convert English measures to equivalent English measures.	29. 1 gal. = ______ qt. = ______ pt. 1 yd. = ______ ft. = ______ in.
30. Add and subtract measures without regrouping.	30. If one board is 2 ft. 3 in. long, and another is 3 ft. 1 in. long, how long are the two boards altogether?
31. Apply the common English measures of length, capacity (dry and liquid), weight, time, money, and counting. Begin to use metric measures of length and mass.	31. a. Coal is measured by the ______. b. A decade is ______ years. c. On many thermometers, each mark represents ______ degrees. d. A__________B The segment from A to B measures ______ centimeters to the nearest centimeter.
32. Measure lengths to the nearest sixteenth-inch and to the nearest millimeter. Measure angles to the nearest degree.	32. A__________B The segment from A to B measures ______ millimeters to the nearest one millimeter.
33. Convert English measures to equivalent English measures.	33. 1 gal. = ______ pt. 1000 lb. = ______ ton.
34. Add, subtract, and multiply measures with simple regrouping.	34. If I saw 2 ft. 8 in. from a board 6 ft. 4 in. long, how long is the piece left?
35. Apply the tables for common English measures of length, weight, capacity, time, counting, area, and volume.	35. 1 sq. ft. = ______ sq. in. 27 cu. ft. = 1 cu. yd. 1 cwt. = ______ lb.
36. Use the common metric units of length, mass, and volume.	36. a. My mother sent me to the store to buy half a kilogram of butter. b. About how many kilometers measure the distance to Indianapolis.

Measurement: Performance Objectives (Continued)

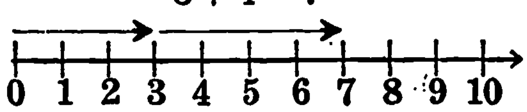
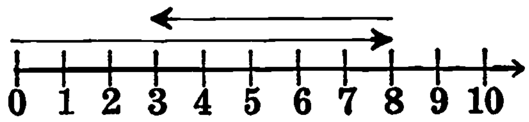
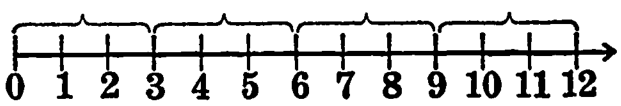
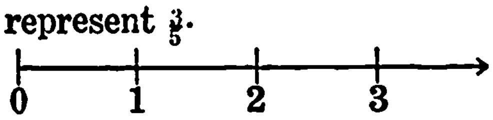
Objectives	Examples
37. Recognize that error is inherent in measurement, and estimate variations caused by error.	37. In measuring a segment precise to the nearest inch, the greatest possible error is ______ inch.
38. Convert English measures to equivalent English measures.	38. 2 lb. 16 oz. = ______ oz. How many fluid ounces are there in 2 qt. 1 pt.? How many cups?
39. Add, subtract, and multiply measures with regrouping.	39. What time would it be 6 hr. from 9:15 A.M.?
40. Utilize the standard measuring devices.	40. a. Measure this angle precise to one degree. b. Use a balance scale to weigh this object.
41. Use the tables for common English and metric measures of length, area, volume, weight (mass), time, money, and temperature.	41. 2×3 ft. 6 in. = ______ in. 1 yd. 3 ft. = ______ in. 2.5 m. = ______ cm.
42. Convert English measures to metric equivalents and conversely.	42. 2.54 cm. = 1 in. 1 km. = ______ mi.
43. Recognize that error is inherent in measurement, and use the concepts of precision and accuracy.	43. Given two measures, 4202 ft. and 35.2 cm., a. Which measure is more precise? b. Which measure is more accurate?
44. Add, subtract, multiply, and divide appropriate measures with regrouping.	44. If I divide $1\frac{1}{2}$ lb. of candy among 4 children, how much should each child get?
45. Use the standard measuring devices.	45. Use tape measure, balance scales, stopwatches, etc.
46. Use the tables for common English and metric measures of length, area, volume, weight (mass), time, money, and temperature.	46. 25 cm. = ______ dm. 1 acre = ______ sq. rd.
47. Convert English measures to metric, and conversely.	47. 1 m. = ______ in. ______ km. = 1 mi.
48. Recognize the existence of error in measurement, and apply the concepts of absolute and relative error.	48. Given two measures, 304 ft. and 151 cm. a. What is the greatest possible error in each measure? b. What is the relative error in each measure. c. What rule-of-thumb are we using when we say the two measures show the same accuracy?
49. Add, subtract, multiply, and divide measures, rounding answers properly.	49. If you add 5.76 cm. to 2.2 cm. what is a reasonable answer?

Note to teachers: This performance objective should generate, on an operational level, several instructional objectives which will have to do, among other things, with some fundamental agreements about significant figures and operations on approximate numbers.

Graphing and Functions: Performance Objectives

Suggested Grade Levels

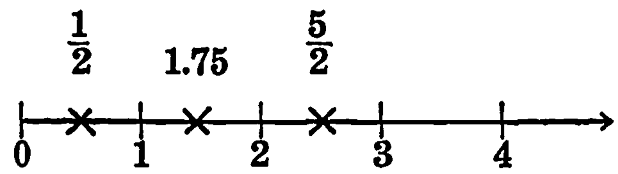
	K	1	2	3	4	5	6	7	8
Items	1	1 to 2	2 to 3	3 to 6	5 to 8	6 to 11	10 to 18	14 to 20	14 to 24

Objectives	Examples
1. Begin use of number line.	1. Make number line on floor with masking tape. Children count "how many from 1 to 10. (Also relate to thermometer.)
2. Use the number line to illustrate an addition or subtraction fact.	2. $3 + 4 = 7$  $8 - 5 = 3$ 
3. List a rate equivalent to a given rate.	3. ○○○ for 2¢ or ○○○○○○ for ______ cents.
4. Use the number line to illustrate a multiplication or division fact.	4. $4 \times 3 = 12$ 
5. Given a rate or a ratio, find another pair of numbers that is equivalent to the given rate.	5. Apples cost 3 for 10 cents. How many can Lisa buy for 30 cents?
6. Find a new number using a given rule.	6. Rule: Multiply the given number by 3 and add 4. E.g., Given: $3 \times (5) + 4$, Result: 19.
7. Solve problems involving readiness activities for integers.	7. Word problems involving thermometer readings. Number line activities using arrows left and right from 0. ($\longleftarrow$ 4) (2 $\longrightarrow$)
8. Find measures of objects when presented with scale drawings (road map, blueprint).	8. Use an Indiana road map to find the mileage between two cities.
9. Make a scale drawing of simple objects (floor plan of a room).	9. Let $\frac{1}{4}$ in. represent 1 ft. Make a scale drawing of a room with no windows which is 16 ft. by 12 ft.
10. Locate points on a number line which correspond to fractions.	10. Locate a point on the line below which would represent $\frac{3}{5}$. 

Graphing and Functions: Performance Objectives (Continued)

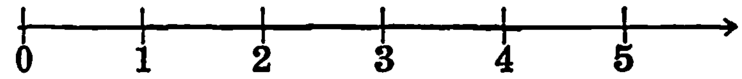
Objectives / **Examples**

11. Construct a number line and locate points which correspond to fractions and decimals.

11. 

12. Indicate the part of the number line which represents the solution set for inequalities and equations.

12. a. Shade the section of the number line below which represents numbers satisfying $X > 3$.



b. Does 3 satisfy $X > 3$? How is this represented on the number line?

c. Give some numbers not represented on the number line which satisfy $X > 3$.

13. Use per cent as a special form of ratio.

13. 20% is a member of the set of ratios where the 2nd term is 100.

$$\left\{\frac{1}{5}, \frac{2}{10}, \ldots, \frac{20}{100}, \frac{21}{105}, \ldots\right\}$$

14. Find the percentage of a number, given the rate and the base.

14. What is 10% of 60? 10% of 60 = $\square$. (6)

15. Find the rate, given the percentage and the base.

15. What per cent of 80 is 5? $\square$% of 80 = 5. ($6\frac{2}{3}$%)

16. Find the base, given the percentage and the per cent.

16. 15 is 10% of what number? 10% of $\square$ = 15. (150)

17. Use a proportion as a means of helping to solve a given problem. m.

17. John's team won 4 out of 16 games. What per cent of the games did John's team win?

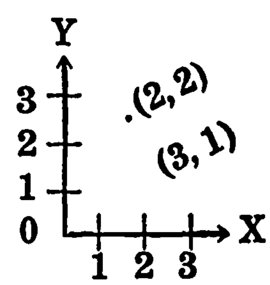
$$\left(\frac{4}{16} = \frac{\square}{100}\right)$$

18. Solve problems using the rates when comparing quantities with unlike units.

18. 6 candy bars cost 30 cents. Rate is $\square$ candy bar per 1 cent or 1 candy bar for 5 cents.

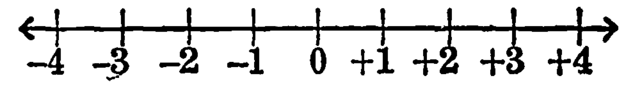
$$\frac{6}{30} = \frac{1}{\square}$$

19. Graph ordered pairs of whole numbers on a coordinate plane.

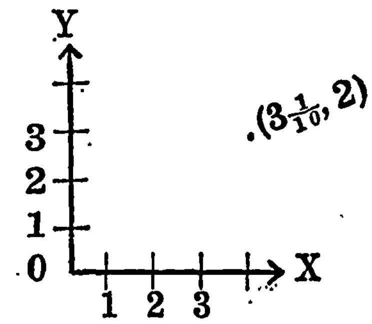
19. 

Objectives	Examples

20. Extend the use of the number line to include negative numbers.

20. 

21. Graph ordered pairs of rational numbers on a coordinatized plane.

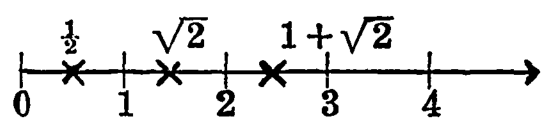
21. 

22. Given the domain of a function and a rule for the function, compute the ordered pairs in the function.

22. $f(x) = 2x + 3$

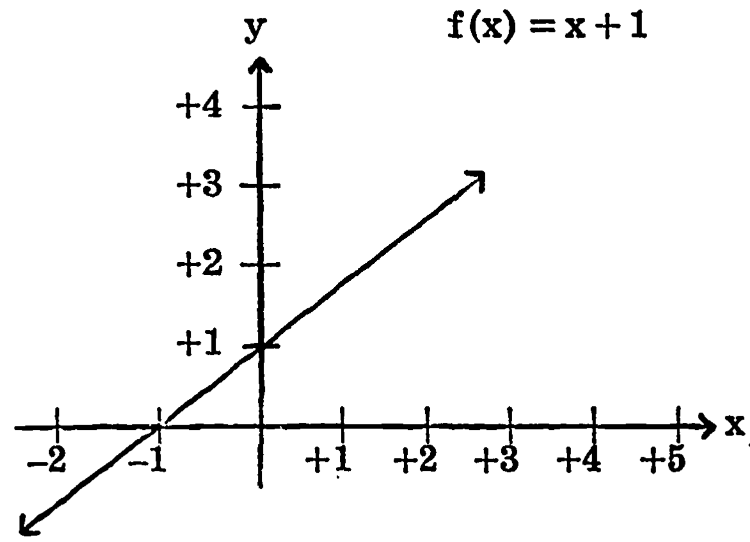
x	f(x)
1	5
2	?
3	9
4	11

23. Graph real numbers on a real number line.

23. 

24. Graph linear function.

24. $f(x) = x + 1$

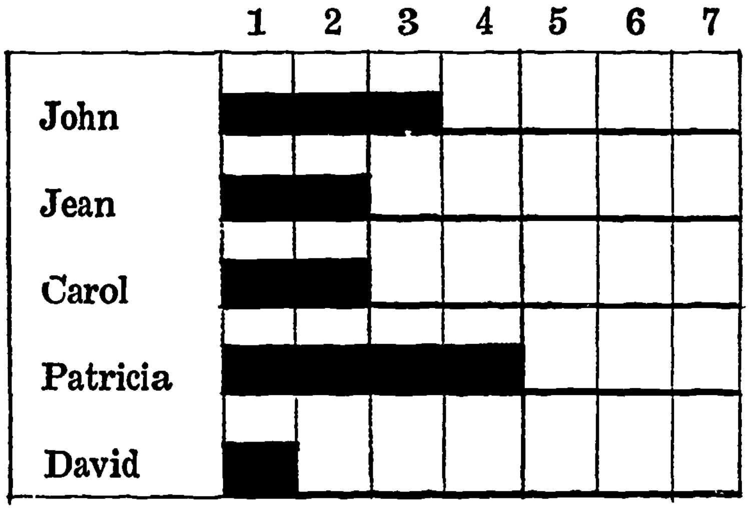


Probability and Statistics: Performance Objectives

Suggested Grade Levels

	K	1	2	3	4	5	6	7	8
Items	1 to 3	1 to 4	3 to 6	4 to 9	7 to 11	9 to 14	11 to 16	13 to 17	15 to 19

Objectives	Examples
1. Make comparisons of likelihood.	1. Children discuss the likelihood of such things as: a. drawing a red marble out of a bag containing a red marble and a green marble. b. a hexagonal spinner's stopping on the red edge if each edge is marked with a different color. c. winning a game with two competitors compared to winning a game with four competitors. d. having snow on Christmas.
2. Gather, organize, and interpret some information on bar graphs.	2. Heights of children are drawn on a sheet of newsprint on the wall, and children discuss ranges, etc., without using technical terms.
3. Predict simple outcomes.	3. If you pick a ball without looking from a bag of 5 green and 1 red, are you more likely to get a green ball or a red ball?—Spinner games.
4. Use average as a number that helps describe a group.	4. Tell the approximate average weight of the children after observing each child being weighed.
5. Interpret the information in a bar graph.	5. Amount of practice in the Smith family (week, hours).



Who practiced the most? The least? How much more did Patricia practice than David? Etc.

Objectives / **Examples**

6. Select the most probable event from a finite sample.

6. Given: a) A bag with 5 green and 4 red marbles, b) a bag with 3 green and 3 red marbles, c) a bag with 5 green, 4 red, and 6 blue. In drawing a marble, which is more likely to be drawn in each of the examples a), b), and c)?

7. Count the number of possible arrangements of a small set of different objects.

7. How many different arrangements in a row can David make of his 4 favorite football players' pictures?

8. Construct a frequency table from simple data, and find the average and range.

8. Measure the height of children in the room. Keep a record and group the heights. Look at the range (shortest — tallest) and the average.

9. Construct and interpret bar and line graphs.

9. Graph mathematics scores, other individual or class scores, weather information for periods of days or weeks.

10. Interpret picture graphs.

10. Amount saved by Joe in a five year period.

1964 $$$$$	1967 $$$$$$$$$$
1965 $$$$$$	1968 $$$$$$$$
1966 $$$	

Each $ represents $10.

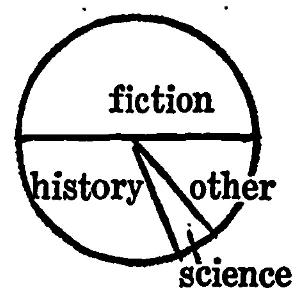
In which year did Joe save the most money? How much? How much did he save in 1965? Etc.

11. Use fractions to represent probabilities.

11. With 6 white balls and 1 red ball the chance of drawing a red ball on first draw is 1–7 or $\frac{1}{7}$.

12. Read and interpret data given in circle graphs.

12.



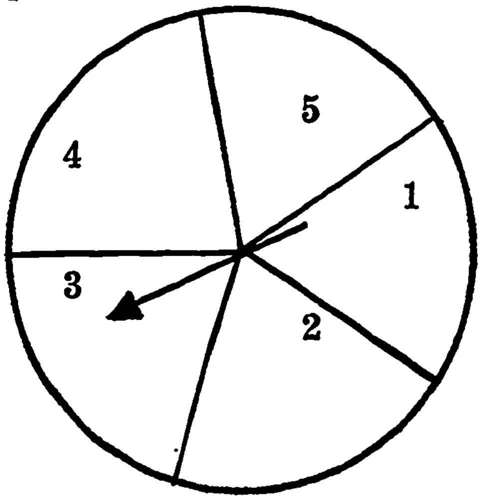
The circle graph shows the incidence of each kind of book in the school library.

13. Count the number of finite arrangements for a set of $\underline{n}$ different numbers.

13. How many ways can a set of 8 elements be arranged in a row?

14. Predict the probability that an event will occur.

14. A box contains 7 red and 5 white balls. What is the probability of your drawing at random one white ball?

Examples	Examples
15. Construct a graph which represents a given set of numerical data.	15. Make a bar graph that shows the results of the first week of the light bulb sale if the green team sold 70 bulbs, the blue team sold 20 bulbs, the red team sold 25 bulbs, and the yellow team sold 85 bulbs.
16. Find the mean and median of a set of numbers.	16. A = {10, 13, 11, 19, 20} The mean of A is 14.6. The median of A is 13.
17. Correctly analyze and solve simple probability problems.	17. A dial is divided into five spaces and a spinning pointer is equally likely to stop in each of the spaces. What is the probability that the pointer will stop in the 5 space? In the 2 or 1 space? 
18. Find the various measures of central tendency of a given set of numerical data.	18. The following is a set of test scores: 65, 67, 74, 74, 75, 76, 16, 76, 78, 78, 84, 87, 90, 94, 96. The mean score is 80. The mode is 76. The median is 77.
19. Analyze and solve basic probability problems.	19. In a toss of two coins, what is the probability of getting a head and a tail? In a toss of a pair of dice, what is the probability the sum of the numbers is eight?

Part III

Secondary Performance Objectives

COURSE TITLES APPROVED BY THE GENERAL COMMISSION

MATHEMATICS

Subjects	Year	Periods Per Week	Semester	Unit Value
Basic Mathematics	9	5	2	1
General Mathematics I (first course)	9–10	5	2	1
General Mathematics II (second course)	10–11	5	2	1
Algebra I (first course)	8–10	5	2	1
Geometry	9–12	5	2	1
Algebra II (second course)	9–12	5	1 or 2	.5 or 1
Mathematics IV				
Trigonometry	11–12	5	1	.5
College Algebra	11–12	5	1	.5
Unified Course	11–12	5	1 or 2	.5 or 1
Business Mathematics	11–12	5	1 or 2	.5 or 1
Shop Mathematics	10–12	5	1 or 2	.5 or 1
Practical Senior Mathematics	12	5	1 or 2	.5 or 1
Analytic Geometry	11–12	5	1	.5
Solid Geometry	11–12	5	1	.5
Calculus	12	5	2	1
Advanced Mathematics	11–12	5	1	.5
Computer Mathematics				
Probability and Statistics				
Modern Abstract Algebra				
Linear Algebra				
Matrix Algebra				

Prior approval for Advanced Mathematics courses is required from the Office of the State Superintendent.

The courses listed are approved for all high schools for the years and credits stated. Other courses may be offered for credit provided a course outline is approved by the Office of the State Superintendent of Public Instruction. The minimum graduation requirement of one unit of mathematics is met by Basic Mathematics, General Mathematics, or Algebra I, but not by combinations of ½ units.

Mature students in grade 8 may be offered Algebra I. However, if the course is taken in grade 8 it does not fulfill the high school graduation requirement of one year of mathematics.

Mathematics IV may be composed of either one semester of Trigonometry and one semester of College Algebra, or two semesters of a unified course which places emphasis on the content of both these subjects.

Sequence Chart

As a department of mathematics endeavors to design a curriculum for its students, several factors must be considered. Among these factors are the goals, achievement, background, and the ability of each student. The existence of variation in these factors among the student population of a school suggests that several different mathematics programs should be made available in order to meet the needs of each student. An indication of how a mathematics program of this type could be established readily within most school systems of the State is displayed in the chart.

The chart identifies three basic sequences of mathematics courses. Provisions have been made for students to be shifted among the various sequences if a change in any of the three factors listed above should occur for any student during his studies in mathematics. The chart is suggestive rather than imperative. It presents a possible framework within which schools may choose to operate. This guide lists the performance objectives of the courses referred to in the chart.

Consider how the chart displays a possible path for a student who has entered Sequence I at Grade 7. The student would pursue a typical program through Grades 7 and 8, probably using standard materials published for a student at this level. In Grade 9 the student would have two options available. Either he could enroll in Algebra I or he could enroll in General Mathematics I. On successful completion of Grade 9, and provided the student desired to continue in mathematics, he may elect any one of four options: a. He may continue in Sequence I to General Mathematics II. b. He may enroll in Business Mathematics. c. He may enroll in Shop Mathematics. d. He may enroll in Algebra I.

All courses are one unit (2 credit) courses except for Practical Senior Mathematics and the course classified as Advanced Mathematics. One-half unit may be offered for one credit of Algebra II, Math IV, Business Mathematics, or Shop Mathematics. In other courses students must have one full year (one unit) for credit.

<table>
<tr><th>Levels</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th></tr>
<tr><td rowspan="2">Sequence 3</td><td rowspan="2">(Enriched) 7-th Grade Math</td><td rowspan="2">Algebra I
(Enriched) 8-th Grade Math</td><td rowspan="2">Geometry
Algebra I</td><td rowspan="2">Algebra II
Geometry</td><td>Math IV*
Algebra II</td><td>Calculus
Math IV*</td></tr>
<tr><td colspan="2">Advanced Math**</td></tr>
<tr><td rowspan="3">Sequence 1</td><td rowspan="3">7-th Grade Math</td><td rowspan="3">8-th Grade Math</td><td>General Math I</td><td>General Math II</td><td colspan="2">Analytic Geometry
Solid Geometry</td></tr>
<tr><td></td><td></td><td colspan="2">Business Math</td></tr>
<tr><td></td><td colspan="3">Shop Math</td></tr>
<tr><td>Sequence 2</td><td>Basic 7 Math</td><td>Basic 8 Math</td><td>Basic Math</td><td colspan="2"></td><td>Practical Senior Math</td></tr>
</table>

* Math IV may be either a unified course or separate semester courses in Trigonometry or College Algebra.

** This name applies to the following courses: Computer Mathematics, Probability and Statistics, Modern Abstract Algebra, Linear Algebra, Matrix Algebra—all ½ unit (1 credit) courses.

Basic Mathematics—Performance Objectives

On completion of the course, the student should be able to:

1. read and write selected numerals (decimal fractions and whole) in both positional and polynomial notation.
2. order selected sets of whole and rational numbers with or without using the number line.
3. recognize and use: a. Properties of one, b. Properties of zero, c. Associative (grouping), d. Commutative (order), e. Closure, f. Inverses (opposites), g. Distributive (multiplication-addition property).
4. use the standard algorithms for the operations of arithmetic as they apply to whole numbers and rational numbers. (This is the major objective of this course.)
5. solve selected simply worded problems.
6. solve simple number sentences of equality (and inequality)*.
7. use measurements taken from ruler, yardsticks, tape, clock and scales in problem situations taken from everyday life.
8. recognize and construct (with protractor and ruler) triangles, circles, squares, and rectangles.
9. compute perimeters and areas of simple plane figures. (See no. 8.)

*10. recognize and sketch rectangular prisms, cylinders, and pyramids.

*11. compute volume and lateral area of figures in no. 10.

12. recognize and work with the following ideas from number theory: a. Odd and Even, b. Divisibility (simple rules), c. Greatest Common Factor, d. Least Common Multiple.
13. apply the concept of ratio and proportion to: a. scaled drawings, b. distance-time-rate, c. percent.
14. collect and classify selected data which apply to common graphs (bar, circle, and line).

Note: Students in this course may be operating at several grade levels below their usual placement.

* Optional

** General Mathematics I (First Course)—Performance Objectives

On completion of the course, the student should be able to:

1. recognize, state, and use the basic properties of the numbers of arithmetic with respect to addition and multiplication, namely: closure, commutativity, associativity, distributivity, identity elements, and inverses.
2. write base ten numerals in either positional or polynomial forms; extend to other number bases with operations optional.
3. name the factors of composite whole numbers less than 100, and identify prime numbers.
4. perform divisibility tests for 2, 3, 4, 5, 6, 9, and 10.
5. find the Greatest Common Factor and Least Common Multiple of pairs of whole numbers each less than 100.
6. identify and illustrate properties of sets including subsets, set membership, and the operations of union and intersection.
7. compute with whole numbers up to 1,000,000 to find sums, differences, products and quotients with reasonable speed and accuracy.
8. compute with rational numbers in simple fraction form to find sums, differences, products and quotients with reasonable speed and accuracy.
9. compute with rational numbers expressed as decimals (at least to 100-ths) to find sums, differences, products, and quotients with reasonable speed and accuracy.
10. convert percents between 10^{-3} and 10^{3} to fraction or decimal form and conversely.
11. express relationships in the form of ratios, and write and solve proportions correctly describing realistic situations.
12. solve simple problems up to and including three steps which involve applications of mathematics to geometry, business, home management, science, and other situations relevant to present environment.
13. prepare and interpret simple statistical graphs of the following form: circle, bar, line, and divided bar.
14. identify and use non-metric relations among points, lines, and planes, specifically: coincidence, parallelism, perpendicularity, intersection.
15. identify and use properties of simple plane figures including lines, line segments, rays, angles, triangles, quadrilaterals, and circles.
16. measure and compute lengths and areas using appropriate formulas for line segments, angles, polygons, and other simple plane figures.
17. construct, using ruler, compass, and protractor, the common plane geometric figures mentioned above.
18. identify and state order relations in the set of integers.

** The objectives of General Mathematics I and General Mathematics II are subject to minor shifting from one course to the other.

19. identify and find the additive inverse of any integer.
20. perform the operations of addition and subtraction on the set of integers with reasonable accuracy.
21. use letters as variables in number expressions or relations.
22. find numbers in a replacement set of a variable which make a sentence involving integers true.
23. find and write the solution set of simple open sentences, primarily equations but including some inequalities.
24. represent solution sets on a number line.
25. find the absolute value of any integer.
26. perform the basic operations with numbers expressed with exponents.
27. state and use the more common units of the metric system of measurement.
28. estimate answers to computational verbal problems with less than 20% error.

General Mathematics II (Second Course)—Performance Objectives

On completion of the course, the student should be able to:

1. recognize, state, and apply the properties of the set of rational numbers for the operations of addition and multiplication, namely: closure, commutativity, associativity, distributivity, identity elements, and inverses.
2. write, using set notation, examples pertaining to the relations of set membership and subsets and the operations of union and intersection.
3. compute sums, differences, products, and quotients with reasonable speed and accuracy using positive rational numbers between .0001 and 1,000,000 written as whole numbers, fractions, or decimals.
4. identify place values in base ten numerals to billions and to ten thousandths.
5. determine the order of rational numbers regardless of the written form.
6. find the prime factors of composite numbers less than 500.
7. find the exact positive square root of perfect square positive integers up to 1,000, and find the approximate square root of irrational numbers less than 1,000 using a table (and by computation)*.
8. solve open sentences involving first and second degree polynomials in one variable (both equalities and inequalities). Write the solution set involving rational numbers only, and check the numbers in the solution set.
9. graph the solution set of equations and inequalities on a number line.
10. solve and check the solution set of simple systems of two linear equations in two variables.
11. apply problem solving techniques to solve realistic problems from business, science, geometry, or home management which involve formulas or equations.
12. write ratios and proportions to describe common relationships, and solve resulting equations when appropriate.
13. use exponents to express powers of numbers, and perform operations with different powers of the same number.
14. find and use the absolute value of any rational number.
15. state and apply the Pythagorean relations to problems involving right triangles.
16. write, simplify, and perform the four basic operations with simple polynomials.
17. simplify and perform the four basic operations with simple rational expressions.

* Optional

18. measure accurately, and interpret and use the results of measurement.

19. construct and analyze simple scale drawings.

20. construct figures, and apply the fundamental properties of congruence and similarity.

21. construct and interpret graphs in two dimensions.

22. solve factorable quadratic equations, and check the solution set.

*23. prepare a series of logical statements which constitute an informal proof of a simple mathematical theorem.

*24. write the definitions of the three basic trigonometric ratios, state and use the fundamental identities, and apply this knowledge to the solution of simple problems.

25. draw "tree" diagrams, and calculate the probability of certain events.

* Optional

Algebra I (First Course) Performance Objectives

On completion of the course, the student should be able to:

1. describe sets using the listing method or set builder (rule) method.
2. perform the operations of union and intersection on sets.
3. use Venn diagrams to associate the universal set with the complement set to and to illustrate other operations on sets.
4. map the set of real numbers onto the number line.
5. identify and state order relations on the set of real numbers.
6. perform the operations of addition, subtraction, multiplication, and division on the set of real numbers.
7. find and use the absolute value of any real number.
8. develop the structure of the real number system concerning the basic properties of addition and multiplication, namely; closure, commutativity, associativity, distributivity, identity elements, and inverses.
9. apply the "order of operations" to a series of indicated operations (computations).
10. evaluate algebraic expressions by direct substitution.
11. solve open linear sentences (equalities and inequalities), and write solutions sets.
12. graph the solution sets of open sentences on a number line.
13. solve linear equations that contain more than one operation, and justify each step.
14. develop other properties of number sets, and prove selected theorems about operations on numbers.
15. extend the basic properties to describe the relationship between addition-subtraction and multiplication-division.
16. analyze verbal problems, translate into mathematical sentences, find the solution set, and check.
17. graph linear equations with two unknowns on a Cartesian coordinate plane.
18. estimate the ordered pair of real numbers in the solution set by graphing a system of two linear equations in two variables on the coordinate plane.
19. state and apply integer (and rational)* exponents, and develop their properties.
20. perform the basic operations on polynomials.
21. find the common solution of a system of linear equations by the subsituation and addition methods (determinants, matrix methods)*.

* Optional

22. perform the "special products" of polynomials, find the "special factors" of polynomials, and factor general trinomials.
23. solve quadratic equations with real roots by factoring, completing the square, and using the quadratic formula methods.
24. use factoring to solve geometry, business, and science problems.
25. extend the fundamental operations to include rational expressions formed by the quotient of polynomials.
26. write and solve equations involving proportions.
27. state and apply the basic properties of irrational numbers.
28. find the approximate square root of a positive real number by an algorithm or by a table.
29. perform the basic operations on the set of irrational numbers. Rationalize numerators and denominators, and express them in simplest radical form.
30. distinguish between relations which are functions and those which are not functions.

*31. solve simple probability problems.

*32. use basic trigonometric relations to solve problems involving right triangles. Include the use of tables.

Geometry—Performance Objectives

On completion of the course, the student should be able to:

1. apply the concept of set notation, including subsets and the operations of union and intersection, to geometric figures.
2. use the basic axioms and theorems regarding operations on real numbers.
3. identify the components of a deductive system: undefined terms, defined terms, assumptions (axioms or postulates), and theorems.
4. recognize inductive reasoning and its relation to everyday living.
5. state the definitions of basic geometric figures such as line segment, skew lines, parallel lines, ray, angle, triangle, parallelogram, trapezoid square, rectangle, regular polygon, etc.
6. recognize and use the concepts of elementary logic to formulate a proof of a theorem.
7. state the meaning of congruence as applied to line segments, angles, triangles, and other geometric figures.
8. write simple proofs applying the basic congruence properties for congruent triangles. (ASA, SAS, SSS)
9. write simple proofs applying the basic congruence theorems specifically applicable to right triangles.
10. state and apply methods of proving lines and planes parallel.
11. state and use conclusions which follow from having lines and/or planes parallel.
12. define similarity for selected geometric figures, prove similarity theorems, and use consequent properties to solve problems.
13. work numerical problems using ratios and proportions, including the geometric mean, related to similar polygons.
14. recognize and use the trigonometric ratios of sine, cosine, tangent as related to the right triangle.
15. state, prove, and use the Pythagorean Theorem in proofs and in applications.
16. define circle, sphere, and related parts: diameter, radius, arc, chord, tangent, secant, segment, and sector.
17. use postulates and theorems pertaining to congruent radii, chords, central angles, and arcs in writing simple proofs concerning circles.

*18. prove and apply the theorems concerning "power of a point" as related to a circle.

19. find areas and perimeters of triangles, trapezoids, parallelograms, rectangles, squares, circles, and regular hexagons and octagons.
20. find volumes of spheres, cones, cylinders, prisms, and pyramids.

* Optional

21. recognize and write indirect proofs.
22. make simple constructions with compass and straight-edge. These should include copying a line segment, copying an angle, the perpendicular bisector of a line segment, bisecting an angle, the perpendicular to a line through a given point, a line parallel to a given line through a point not on the given line, dividing a given segment into a specified number of congruent segments.
23. use a protractor to "measure" angles.
24. state and use the special properties of isosceles triangles.
25. state and use the special properties of equilateral triangles.
26. describe and locate by construction methods, the incenter, circumcenter, centroid, and orthocenter of triangles.
*27. characterize and locate specific sets of points such as: points a given distance from a given point, points a given distance from a given line, points equidistant from the sides of a given angle.
28. find area of spheres and total and lateral areas of cones, cylinders, prisms, and pyramids.
*29. use coordinate geometry to describe points and lines in a plane.
30. state and use the basic inequalities relating to sides and angles of triangles.
31. state the converse, inverse, and contrapositive of a theorem; and identify equivalent statements.
32. recognize and illustrate dihedral and polyhedral angles.
*33. investigate other geometries such as Non-Euclidean geometry.

* Optional

Algebra II (Second Course)—Performance Objectives

On completion of the course, the student should be able to:

1. state and illustrate the properties of the natural numbers—integers, rationals, reals, and complex; and identify their internal structure.
*2. identify fundamental set concepts and operations on sets with appropriate symbols.
3. perform the fundamental operations on algebraic expressions.
4. find solution sets to open sentences of one variable, and graph on the number line.
5. state the interrelationships among the subsets of the real numbers.
6. show the one-to-one correspondence between the points of the cartesian plane and the infinite set of ordered pairs of real numbers. (Cartesian Product $R \times R$)
7. symbolize and graph selected relations.
8. identify and graph the inverse of a given relation.
9. identify the functions from a list of relations.
10. construct composite, sum, and product functions from given functions.
11. construct the inverse of a function, and determine whether it is a function.
12. recognize and construct special functions: constant, absolute value, greatest integer, etc.
13. describe an increasing (decreasing) function, and tell intuitively whether it is continuous.
14. determine the distance between two points in a plane, and find the coordinates of the midpoint of the segment.
15. given sufficient conditions, write the equation of a line using standard forms: intercept, two-point, etc.
16. describe the structure of the system of polynomials, and perform the fundamental operations.
17. factor polynomials over selected number systems.
18. perform synthetic division; and use the Remainder Theorem, the Factor Theorem, and the Fundamental Theorem of Algebra.
19. determine the rational roots of selected polynomial equations of degree greater than two.
20. graph the quadratic function, determine its domain and range, find the zeros, locate axis of symmetry, and determine absolute maximum (minimum) values.
21. relate the sum and difference of the roots of a quadratic equation to the coefficients, and determine their number and nature.

* Optional

22. describe* the structure of the system of rational expressions, and perform the fundamental operations. (P/Q, $Q \neq O$, P and Q are polynomials.)

23. derive the equations of the conics and define associated terms: focus, directrix, vertex, and asymptotes. Graph each conic and translate to a new coordinate system.

24. define and show equivalent systems of equations. Solve systems of equations in two variables by graphing, addition, substitution, determinants*, and matrices*.

25. solve systems of linear-quadratic and bi-quadratic equations and inequalities.

*26. coordinatize space with ordered triples of real numbers, calculate distance, determine the equation of a plane, and plot its trace.

27. solve systems of three equations in three variables.

28. extend the fundamental laws of exponents to include integral, rational, and rational approximations to real exponents.

29. solve selected exponential equations, and graph selected exponential functions.

30. use the fundamental laws of logarithms, compute selected arithmetic expressions, solve simple logarithmic equations, and graph selected logarithmic functions.

31. evaluate selected finite arithmetic, geometric, and harmonic series. Find the limit of an infinite series if the limit exists.

*32. write the distinguishing properties of a mathematical system, e.g., a group, field.

*33. write compound statements in English or in symbols using the logical connectives of negation, conjunction, inclusive and exclusive disjunction, related and varied forms of the conditional and biconditional.

*34. identify a compound statement as a tautology or a contradiction through the construction of truth tables or logical argument.

*35. utilize the quantifiers "all", "some", and "none" in the analysis of sentence structure.

*36. use the properties of discreteness, density, unique factorization, and Archimedean in describing subsets of the real numbers. Prove selected theorems using these properties.

*37. calculate the permutations (and combinations) of n things taken r at a time.

38. use the Binomial Theorem to expand a binomial raised to the nth power and calculate the rth term.

* Optional

**39. demonstrate the "wrapping" function that maps the real numbers into the set of ordered pairs of real numbers which identify the points of the unit circle.

**40. derive the circular (trigonometric) functions from the concept of the wrapping function.

**41. recognize and use the periodic nature of the circular functions.

**42. relate radian measure of an angle to the measure of a path on a circle, and evaluate the circular functions of real numbers using this concept.

**43. derive the Pythagorean relationships and the basic trigonometric identities.

**44. derive the inverses of the trigonometric functions, and limit the domain so that the inverses are functions.

**45. solve open sentences involving the trigonometric functions.

**46. sketch the graphs of the circular functions and their inverses.

**47. apply the trigonometric functions to solution of selected problems.

** If there is a separate course in Trigonometry, teachers may want to defer these items.

Mathematics IV (Unified Course**)—Perfomance Objectives

On completion of the course, the student should be able to:

These objectives are often met in the second course in algebra. However, it may be necessary to review and extend them in this course.

1. state the definitions of, write symbols which represent, and identify given symbols as representing real numbers, complex numbers, rational numbers, irrational numbers, and integers.
2. state and illustrate, using set notation, the meaning of equivalent sets, subsets, universal set, quantifiers, and mappings.
3. use Venn diagrams to show the set operations of union, intersection, and complement.
4. state the field postulates, list which of the sets of numbers in objective 1 satisfy the postulates, and state and illustrate postulates which the other sets of numbers do not satisfy.
5. prove the existence of irrational numbers.
6. map the ordered pairs of real numbers $R \times R$ onto the Cartesian plane.
7. calculate the distance between any two points.
8. define relation and function, and give illustrations.
9. define and state the domain and range of polynomial, rational, algebraic, and special functions.
10. determine the sum, difference, product, quotient, and composition of two functions; and give the domain for each.
11. determine the relation which is the inverse of a given function, determine if this relation is a function, and produce examples of inverse functions.
12. sketch the graphs of polynomial, rational, algebraic, absolute value, and greatest integer functions.
13. solve both linear and quadratic open sentences in one variable (equalities and inequalities), and graph the solution sets on the number line.
14. solve both linear and quadratic open sentences in two variables, and graph the solution sets on the coordinate plane.
15. define absolute value, and solve and graph open sentences involving absolute value.
16. write the defining equation for the exponential function, state the domain and range, and graph the exponential function for various permissible values of the base.
17. write the defining logarithmic equation equivalent to a given exponential function, state the domain and range, and graph the logarithmic function for various permissible values of the base.
18. solve exponential and logarithmic equations, and apply to the solution of problems.

** (See footnote for trigonometry and college algebra pages.)

19. perform computation using tabulated values and the logarithmic properties for products, quotients, powers, and change of base.

20. solve problems involving direct, inverse, and joint variation.

21. use synthetic division to divide a polynomial $P(x)$ by a linear polynomial.

22. state, prove, and use the Remainder, Factor, and Converse of Factor theorems.

23. distinguish between polynomials and polynomial equations, and find the zeros and solution sets respectively.

24. indicate the number of roots of real polynomial equations, and form real polynomial equations using given roots.

*25. state and use DesCartes' Rule of Signs.

26. determine the rational roots of polynomial equations.

27. approximate real zeros of polynomials with real coefficients.

28. state the definitions of the six circular (trigonometric) functions derived from the "wrapping" function on the unit circle.

29. derive, state, and use the fundamental trigonometric relations in proving trigonometric identities.

30. convert degree measure to radian measure of an angle and vice versa.

31. given the radius r of a circle, use the formula $S = r\theta$ to calculate the arc length s subtended by a given central angle θ. Calculate the area of the circular sector and segment.

32. give the values of the trigonometric functions of 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $2\pi/3$, $3\pi/4$, $5\pi/6$, π, $7\pi/6$, $5\pi/4$, $4\pi/3$, $3\pi/2$, $5\pi/3$, $7\pi/4$, $11\pi/6$ and 2π without reference to tables.

33. solve equations for all x in a given interval of the types: i) $a \cdot t\,(nx) = b$ where t is any trigonometric function and ii) quadratic equations solvable by factoring (including common factor) and the formula.

34. state the domain and range for each of the trigonometric functions.

35. define periodic function, period, and amplitude.

36. sketch the graphs, over the interval $(-2\pi, 2\pi)$, of $y = a \sin (bx + c)$ and $y = a \cos (bx + c)$.

37. give the period and amplitude, if defined, for $y = a \sin (bx)$, $y = a \cos (bx)$, and $y = a \tan (bx)$.

38. define, state, and use the formulas for $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$, $\cos(A-B)$, $\sin 2A$, $\cos 2A$, $\sin A/2$, $\cos A/2$ in solving identities and equations. (Include tangent functions.)*

39. define and graph the inverses of the circular functions. Limit the domain of the inverses so that they are functions, and evaluate expressions involving these.

* Optional

40. solve equations using the inverse circular functions.
41. evaluate the circular functions using tables for any permissible real replacement of the variable.
42. solve (including logarithmic solution) for the missing parts of triangles given ASA, SSS, SAS, SSA by using the Law of Sines, Law of Cosines. (Include the Law of Tangents.)*
43. solve applications (including use of vectors) by means of the Law of Sines, the Law of Cosines, and right triangle relations.
44. state and use the rule for equality of complex numbers, and perform the four fundamental operations on complex numbers in either $a + bi$ form or (a, b) form.
45. state the Principle of Mathematical Induction, and prove selected theorems by using mathematical induction.
46. construct the polar coordinate system in a plane, and determine the polar coordinates of any point.
47. give both the rectangular and polar form of a complex number. (Include exponential form.)*
48. find the product and quotient of two complex numbers using the polar form relations.
49. state and use DeMoivre's Theorem. (Prove the theorem by mathematical induction.)*
50. calculate the nth roots of a complex number.
51. solve equations involving the properties and operations of complex numbers.
52. use the Binomal Theorem to write the expansion of $(a + b)^n$ where n is an integer and write the rth term of the expansion. (Prove the Binomial Theorem.)*
53. use the Binomial Theorem to write the first k terms in the expansion of a binomial when n is a non-integer rational number.
54. determine $P(n, r)$ for $r \leqslant n$, circular permutations, and $C(n, r)$ for $r \leqslant n$.
55. determine the probability of success of a single trial if there are n equally likely outcomes.
56. determine the probability of occurrence of two or more independent events, dependent events, mutually exclusively events.
57. determine the probability of an event happening exactly r times in n trials, and at least r times in n trials.
58. define the following: sequence, series, arithmetic series (sequence), geometric series (sequence), harmonic series (sequence).
59. write the terms of a sequence from the recursion formula.
60. use and expand expressions involving the Σ and Π* notations.

* Optional

61. determine the nth term of selected arithmetic and geometric sequences, and determine the sum of a finite number of terms of selected arithmetic and geometric series. Find the sum of convergent infinite geometric series.
62. define matrix; define vector and determinant in terms of a matrix.
63. calculate the sum of two vectors, the product of a scalar and a vector, the inner (dot) and outer products of two vectors, and the norm (length) of a vector. Graph on the complex plane.
64. state and illustrate the properties of the algebra of two-space and three-space vectors.
65. find the sum of two matrices, the product of a scalar and a matrix, and the product of two matrices.
66. state and illustrate the group properties of matrix algebra.
67. evaluate second, third, and fourth order determinants by using the properties of equivalent determinants and/or expansion by minors.
68. find the multiplicative and additive inverses of a square matrix.
69. solve systems of equation using matrices, determinants in addition to algebraic methods.
70. define slope of a line; determine the slope of a line and the related angle of inclination.
71. write the equation of a line which i) contains two given points, ii) contains one given point and has a given slope, iii) is parallel to a given line, iv) is perpendicular to a given line.
72. determine the coordinates of the midpoint of a given segment.
73. find the distance between a point and a line.
74. calculate the angle between two lines.
75. define and write the equations of the tangent and normal to a curve at a given point.
76. graph a curve given the parametric equations of the curve.
77. define and illustrate the conic sections (circle, ellipse, parabola, hyperbola, and degenerate cases).
78. write the equations of the conics when the center (or vertex) is at the origin.
79. define and use the special points and lines associated with the conics (focus, directrix, major axis, eccentricity, asymptotes, etc.).
80. simplify equations of conics by translation of axes.
81. sketch the graph of any equation of the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ (A, C, D, E, and F are real numbers and not both A and C equal 0).
*82. simplify equations of the conics by rotation of axes.
83. prove theorems involving points, lines, and the conics.
84. write the equation of a conic, given sufficient constraints.
85. apply the relations about conics to the solutions of problems.
86. write equations of lines and planes in three-space.

* Optional

Mathematics IV—Trigonometry**—Performance Objectives

Refer to the performance objectives for the Unified Course (pages 77-80). The objectives there numbered 1 through 4, 6 through 8, 28 through 43, and 46 through 51 are appropriate ones for a one semester trigonometry course.

Mathematics IV—College Algebra**—Performance Objectives

Refer to the performance objectives for the Unified Course (pages 77-80). The objectives there numbered 1 through 27, 44, 45, 49 if not done in trigonometry, and 52 through 86 are appropriate ones for a separate college algebra course. Not all of these objectives can be covered in a one semester course. Appropriate objectives for a one-semester course will be determined by the local course of study and the other mathematics courses offered.

** Derivations and proofs are an important and necessary part of this course. Not only should derivations and proofs be presented in the development of topics but students should be able to do some of them. Some derivations and proofs are included in this list of performance objectives. However, this does not imply that these are the only ones to be emphasized. The local course of study will determine the derivations and proofs required of the student.

Business Mathematics—Performance Objectives

On completion of the course, the student should be able to:

1. compute sums, differences, products, and quotients of whole numbers using five-digit numbers and five columns in addition, three-digit numbers in multiplication and division, and five-digit numbers in subtraction.
2. perform the four basic operations with fractions with reasonable speed and accuracy.
3. compute mentally multiplication and division by 10, 100, and 1,000 and the cost of articles purchased in quantities of 10, 100, 1,000; articles priced at 10¢, $1, $10, etc.; articles priced per C, Cw, M, or T.
4. reconcile a bank balance, personal or business; complete sales slips, including computation of a sales tax and federal excise tax; and compute aliquot parts, with absolute accuracy.
5. determine wage income from hourly wages consisting of straight time, overtime, and deductions, including income tax, FICA tax, and miscellaneous.
6. recognize and apply percentage relationships to solve problems.
7. determine income from commissions and commissions plus salary.
8. compute interest by using 60-day method and aliquot parts of 60-days and interpret interest tables.
9. calculate the interest charges of installment buying.
10. determine costs of life, automobile, and fire insurance.
11. read electric, gas, and water meters; and determine the bills.
12. figure costs of transportation of goods by parcel post, express, and freight.
13. compute property, federal income, and state gross income taxes.
14. prepare and use charts that picture the results of business operations.
15. compute trade, single rate, and cash discounts.
16. convert a denominate number to units of lower or higher denomination in both the English and Metric systems.
17. convert measurements from English to Metric system and vice versa.
18. find perimeters and areas of rectangles, triangles, and circles.
19. find areas and volumes of rectangular solids, cubes, and cylinders.

*20. interpret binary numerals, and perform the four operations with binary numerals.

*21. solve problems in probability involving the basic ideas of random choice.

* Optional

Shop Mathematics—Performance Objectives

On completion of the course, the student should be able to:

1. perform the fundamental operations on integers, rational numbers, and real numbers.
2. compute least common multiples and greatest common divisors for selected sets of whole numbers.
3. use basic formulas for computing linear, area, and volume measures of various geometric figures.
4. apply the ideas of ratio and proportion to help solve shop problems.
5. write selected equivalent fractions, decimals, and percents, e.g., $\frac{3}{5} = .6 = 60\%$.
6. solve simple linear equations in one unknown as they arise in problems related to Ohm's Law, surface and volume measure, etc.
7. apply the Pythagorean Theorem to the solution of shop problems.
8. change from one unit of measurement to another both within and between systems.
9. use the properties of geometric figures to help solve problems related to layout, sketching, etc.
10. apply the ideas of percent to: tolerances, efficiencies, ratings, profit, etc.
11. apply trigonometric functions of an acute right triangle to solve shop problems.
12. read and use Mathematical Tables: squares, cubes, square root, decimal equivalents, trigonometric, etc.
13. prepare estimates of materials, costs, etc. for shop projects.

Note: Shop mathematics is a one- or two-semester course which is designed for students in Industrial and/or Vocational Education programs. The main purpose of the course is to help the student to acquire the kinds of mathematical concepts and skills which will help him solve problems that arise in Industrial and/or Vocational shops.

Practical Senior Mathematics—Performance Objectives

**On completion of the course, the student should be able to:

1. compute with reasonable accuracy sums, products, differences, and quotients of whole numbers.
2. multiply numbers in decimal notation mentally by powers of ten.
3. write the prime factorization of whole numbers less than one hundred.
4. write the equivalent forms of fractions, decimals, and percent to represent a number.
5. use ratios and proportions in the solution of problems.
6. apply the idea of percent to solve problems from business or home management.
7. correctly fill out income tax forms (like 1040 or 1040A), given a realistic set of data.
8. compare the cost of borrowing money, given the type of rates charged by credit unions, banks, and other financial organizations.
9. estimate and calculate the cost of installment buying at a given rate of interest.
10. compare the advantages and disadvantages of various forms of investments.
11. estimate and calculate the costs of buying and maintaining a home.
12. estimate and calculate the costs of buying and operating an automobile.
13. find the measure of a given object (length, area, volume, weight, etc.)
14. make basic compass-straightedge constructions.
15. state and use formulas for finding the perimeter, area, and volume of common geometric figures.
16. apply the Pythagorean theorem in solving problems about right triangles.
17. apply geometric and other formulas in solving realistic problems from business and home management.
18. state the purposes of the major types of life insurance policies.
19. state the purposes and approximate the costs and benefits of social security and medical insurance.
20. interpret and construct line, bar, and circle graphs.
21. solve linear equations.
22. use the tangent, sine, and cosine ratios to solve physical problems.

** The objectives of this course will vary considerably with the ability and interests and needs of the students.

Analytic Geometry—Performance Objectives

On completion of the course, the student should be able to:

1. utilize Cartesian Coordinate system for naming points in a plane.
2. compute the distance between any two points in a plane.
3. determine the coordinates of a point that divides a segment into parts whose ratio is given.
4. determine the slope of a line and the related angle of inclination.
5. find the slope of a line perpendicular to a given line.
6. determine the equation of a line which (a) is parallel to either axis, (b) passes through two given points, (c) passes through a given point and has a given slope.
7. associate a given line with an equation of the form $Ax + By + C = 0$ (A, B, and C real numbers and not both A and B equal 0).
8. determine a specified member from a given family of lines.
9. determine (a) existing intercepts, (b) symmetry relative to axes and origin, (c) horizontal and vertical asymptotes, (d) excluded regions, given an equation expressed in rectangular or polar coordinates.
10. discuss the nature of the graph of $y = ax^n$ for replacements of a by real numbers and replacements of n by rational numbers.
11. translate the origin to a new position (a, b) in the plane by using the translation equations $x = x' + a$ and $y = y' + b$.
12. sketch the graph of any equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$ (A, B, C, D, and E real numbers and not both A and B equal 0).
13. describe each conic section as the intersection of a plane and a conical surface.
14. use the general locus definitions to derive the equations of each of the conic sections and define associated terms (e.g., directrix, focus, eccentricity, asymptotes).
15. rotate a given coordinate axis system through a given angle.
16. determine which of the conic sections is represented by the general equation of the conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, given the constants.
17. rotate the axes to remove the product term from the general equation of the conic.
18. recognize equations of conics in polar coordinate form.
19. write the parametric equations of a line.
20. sketch the graph of a function expressed by parametric equations.
21. utilize space coordinates in graphing.
22. sketch the graph of cylinders, cones, and conicoids.
23. sketch the graph of the intersection of two surfaces.
24. define a surface of revolution.
25. apply analytical methods to the proofs of theorems.

Solid Geometry—Performance Objectives

On completion of the course, the student should be able to:

1. define space, surface, vertices, edges, projection, dihedral and polyhedral angles, polyhedrons, spherical polygons.
2. represent spatial relationships in perspective, e.g., perpendicular and oblique lines to a plane, perpendicular, oblique, and parallel planes, geometric solids (prism, pyramid, cylinder, cone, sphere, regular polyhedrons), and spherical polygons.
3. identify and compute the volumes and surface areas of solids listed in no. 2.
4. develop formulas for the volumes and areas of prisms (oblique and right), cylinders, pyramids, cones, and spheres.
5. apply projections of line segments on a plane to solve problems.
6. define locus in space, and solve selected loci problems involving compound loci.
7. perform the fundamental operations on approximate numbers.
8. recognize equivalent and congruent solids.
9. apply these two theorems on limits in the proofs of theorems on the mensuration of the cylinder, cone, and sphere: if $x \to k$, then $cx \to ck$ and if $x = y$, as $x \to m$ and $y \to n$, then $m = n$.
10. define, illustrate, and use spherical distance, polar distance, spherical angle, zone, spherical sector, spherical segment, spherical cone in the solution of problems.
11. state the relationships among the sphere, cone, cylinder, and cube.
12. state and apply Euler's and Cavalieri's theorems.
13. use logarithms in the solutions of applications of volumes and areas of solids and spherical triangles.
14. solve problems involving use of rectangular coordinates in space.
15. identify spatial relationships by recognizing the inter-relatedness of pieces of spatial figures which have been dismantled. Fit them back together again.
17. prove selected theorems by using converses, inverses, contra-positives.
18. recognize the five conic sections, and use cross sections to solve practical problems.
19. define symmetry, and describe the symmetry of selected spatial figures.
20. recognize and classify spatial transformations.

Calculus—Performance Objectives

On completion of the course, the student should be able to:

1. define function, and state the domain and range of a function. Illustrate various methods of combining functions.
2. completely analyze the properties of the conic sections, write the general equations of the conic sections using the definitions or standard forms, and write basic geometric proofs using analytic geometry.
3. write the equation of a line using the two-point form, point-slope form, intercept form, or slope-intercept form; and express in the standard form, $Ax + By + C = 0$.
4. graph functions and relations using the concepts of symmetry, asymptotes, domain, range, translation of axes, and rotation of axes.
5. find the angle between two intersecting lines, and find the distance from a point to a line.
6. find the limit at a finite point and at infinity, if either or both exist, of selected algebraic functions using the epsilon-delta approach.
7. use L'Hospital's rule, if applicable, to find the limit, if it exists, of a function which is discontinuous at a point.
8. apply theorems on limits to find the limit at a finite point or at infinity, if either or both exist, of selected functions.
9. prove $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
10. use the definition of a derivative to find the derivative of selected algebraic functions.
11. use the sum, difference, product, quotient, and chain rule to find the derivative of algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, and (hyperbolic functions)*.
12. use the derivatives of $f(x) = \sin x$ and $g(x) = \cos x$ to derive the derivatives of sec x, csc x, tan x, and cot x.
13. use the derivative to find the maximum, minimum, and points of inflection of the graph of a function.
14. use the derivative to find the equation of the tangent and normal line to a curve at a point on the curve.
15. use the derivative to find the equation of the tangent lines from an external point to a curve.
16. use the derivative to find velocity, acceleration, and solve related rate problems.
17. use the derivative to find the angle between the graphs of two curves.
18. use the derivative to approximate roots of an equation.

* Optional

19. find and apply the derivative of implicit functions and equations written in parametric form.
20. use the methods of substitution, partial fractions, and integration by parts to find the integral of algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, and (hyperbolic functions)*.
21. use the definite integral to find area under a curve and between two curves, length of an arc, surfaces of revolution, and centroids.
22. use the definite integral to find volumes of solids of revolution, find volume by the cylindrical shell and disk methods, and find volume by the method of slicing.
23. apply the integral to selected problems of physics such as hydrostatic force, work, acceleration, and velocity.
24. use the trapezoidal rule to approximate selected definite integrals.
25. evaluate integrals which are infinite or discontinuous.
26. state and illustrate the basic theorems such as the fundamental theorems of differential and integral calculus, mean value theorems of differential and integral calculus, squeeze theorem, and theorems on continuous functions.

*27. prove the theorems stated and implied in no. 26.

*28. Evaluate selected definite integrals using Simpson's rule.

*29. Find and apply the derivative of vector functions.

*30. Find and apply the derivative of equations written in polar co-ordinate form.

Derivations and proofs are an important and necessary part of this course. Not only should derivations and proofs be presented in the development of topics but students should be able to do some of them. Some derivations and proofs are included in this list of performance objectives. However, this does not imply that these are the only ones to be emphasized. The local course of study will determine the derivations and proofs required of the student.

* Optional

**Advanced Mathematics—Computer Mathematics
Performance Objectives

On completion of the course, the student should be able to:

1. display a knowledge of the history of computer science by writing an acceptable brief essay. Do the same for the role of the computer in modern society.
2. do program analyses of selected problems by constructing flow charts.
3. interpret computer solutions with special attention given to order, sequence, and detail.
4. design algorithms for selected problems, and then program the algorithms for the computer.
5. operate a computer or a teletypewriter sender-receiver terminal, and use a variety of input-output devices.
6. develop a proficiency in some compiler language, such as Basic or Fortran IV, in order to write and prepare programs to solve a variety of mathematical and scientific problems.
7. rewrite programs which are rejected.
8. operate related equipment, such as the teletypewriter, interpreter-translator, sorter, etc., necessary to process programs.
9. discriminate among the results of the output, and choose what is significant.
10. exhibit a knowledge of the career opportunities that exist in the computer science field by writing an acceptable essay.
11. list the limitations and strengths of the computer by noting what it can and what it cannot do.
12. perform analyses of functions by supplying equations or ordered pairs to the computer. Observe and interpret the results in graphical form from the teletype printout.
13. write increasingly complex programs for problems pertaining to other courses.

** Prior approval for Advanced Mathematics courses is required from the Office of the State Superintendent.

**Advanced Mathematics—Probability and Statistics Performance Objectives

On completion of the course, the student should be able to:

1. interpret data presented in tabular and graphical form.
2. organize and present data in tubular and graphical form. For example: bar graph, broken line graph, curve line graph, circle graph, pictorial chart, histogram, frequency polygon, and cumulative frequency polygon.
3. summarize and analyze selected data by calculating the measures of central tendency (mean, median, mode); discuss their relative merits.
4. calculate measures of dispersions: range, quartile deviation, variance, and standard deviation.
5. compute measures of central tendency and measures of dispersion from ungrouped data.
6. compute the mean and standard deviation for large numbers of measurements by grouping techniques.
7. determine whether outcomes of an experiment are equally likely.
8. determine whether outcomes of an experiment are mutually exclusive.
9. determine the sample spaces of various experiments.
10. express events as subsets of a sample space.
11. determine whether two events are complementary.
12. determine whether events are exhaustive.
13. develop and apply the formulas for calculating the number of permutations and combinations of n objects taken r at a time, and determine whether the problem involves permutations or combinations.
14. apply correctly the two basic counting principles to determine the number of outcomes in event "A or B" and event "A and B."
15. determine the probability of an event in a finite sample space using the classical definition of theoretical probability.
16. estimate the probability of an event by the relative frequency concept in a series of experiments.
17. apply combinatorial theory to calculate the probability of events.
18. identify binomial experiments and apply the laws of chance to the binomial distribution.
19. compute the conditional probability of event A given B.
20. determine whether two events are independent.

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21. relate the probability of occurrence of two independent events to the probability of their intersection and apply to appropriate problems.
22. compute the mathematical expectation of an event from an actual or theoretical experiment.
23. apply the appropriate additive and multiplicative theorems to determine the probability of multiple events.
24. distinguish between samples and populations and between statistics and parameters.
25. use elementary sampling theory to perform a random sampling.
26. apply the theory of probability to acceptance sampling.
27. apply the theory of probability to test statistical hypotheses involving normal and binomial distributions.
28. use samples to make estimates of population measures.

**Advanced Mathematics—Modern Abstract Algebra
Performance Objectives

On completion of the course, the student should be able to:

1. describe sets using roster or rule notation.
2. perform the operations of union, intersection, Cartesian Product, and relative complement on two sets.
3. illustrate relations and mappings as subsets of Cartesian Products.
4. define and illustrate an equivalence relation.
5. determine the equivalence classes associated with an equivalence relation.
6. define a binary operation as a mapping from $S \times S$ into S.
7. cite the Peano Axioms, and explain why each is necessary to characterize the natural numbers.
8. define groups and semi-groups, and prove selected consequent theorems.
9. define Rings and Integral Domains, and prove selected consequent theorems.
10. define Field, and prove selected consequent theorems.
11. define Isomorphism, and show two groups isomorphic.
12. construct the set of integers from equivalence classes of ordered pairs of natural numbers.
13. show that addition and multiplication of integers are well-defined.
14. construct the set of rational numbers from equivalence classes of ordered pairs of integers.
15. show that addition and multiplication of rational numbers are well-defined.
16. illustrate subsets of the rational numbers which are groups, rings, fields.

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****Advanced Mathematics—Linear Algebra—Performance Objectives**

On completion of the course, the student should be able to:

1. define vector.
2. perform addition and scalar multiplication on vectors.
3. determine if a given system is a vector space.
4. test the definition of a vector space in E_1, E_2, and E_3.
5. test the following properties of vector spaces in E_1, E_2, E_3:
   a. Scalar multiplication of a vector by zero gives the zero vector.
   b. Scalar multiplication of vector V by (–1) gives –V.
   c. The additive Cancellation Law for addition of vectors.
   d. The multiplicative Cancellation Law for scalar multiplication of vectors.
6. test a given system for the definition of a subspace.
7. determine the types of subspaces in E_1, E_2, E_3.
8. show that the intersection of subspaces is a subspace.
9. determine when a set of vectors forms a base for a vector space.
10. provide bases for vector spaces in E_1, E_2, E_3.
11. determine when a transformation in a vector space is linear.
12. recognize and test types of linear transformations in E_1, E_2, E_3.
13. define matrix.
14. add and multiply matrices.
15. multiply matrices by scalars.
16. define equality of matrices.
17. determine when a system of matrices and scalars forms a vector space.
18. verify the properties of matrix addition, scalar multiplication, and matrix multiplication.

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****Advanced Mathematics—Matrix Algebra—Performance Objectives**

On completion of the course, the student should be able to:

1. define a matrix.
2. define equality of matrices.
3. perform matrix multiplication and addition.
4. multiply matrices by scalars.
5. verify the properties of matrix addition, scalar multiplication, and matrix multiplicaton.
6. determine when a matrix has a multiplicative inverse, and compute the inverse when appropriate.
7. compute the determinant of a square matrix by using: a. diagonal method, b. triangulation, c. reduction of order, d. expansion by minors.
8. apply determinants to practical problems such as finding the area of a triangle, given the cartesian coordinates of its vertices or solving a system of n equation in n unknowns.
9. solve linear transformation problems using matrices: e.g., translations and rotations in the cartesian plane.
10. interpret vectors as matrices.
11. use matrices to interpret the properties of parallelism and perpendicularity of vectors in E_2 and E_3.
12. interpret complex numbers as matrices.
13. interpret quaternions as matrices.
14. interpret particular sets of matrices as vector spaces.
15. determine when a set of matrices with the operations of addition and scalar multiplication forms a vector space.

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Part IV

Implementing the Guide

SUGGESTIONS FOR IMPLEMENTING THE GUIDE

The eventual worth of this guide will be determined by the extent of its use in the schools of Indiana. While it is not intended to be prescriptive or restrictive, the suggestions and objectives should not be dismissed lightly at any level; for the intent of the Advisory Committee was to provide a publication which would be helpful to students, teachers, and administrators in a variety of ways. The publication was constructed with the firm belief that students will learn more if they (and we) know what is expected.

Consider the sets of performance objectives. Any one of them might provide a basis on which to construct diagnostic pupil progress charts. Proper use of these charts should promote individualized instruction directed toward meeting actual student needs.

For the classroom teacher whose school has not provided him with a local curriculum guide, this publication provides a wealth of ideas to consider as he prepares his instructional program. The Advisory Committee's intent was to design each section, from the statement of philosophy to the section on textbook selection and bibliography, for maximum use by the classroom teacher.

School superintendents and building principals should find this publication helpful as they consider the overall direction of the instructional program in mathematics. Identification of instructional goals, a continuing prerequisite in planning regular or innovative programs, should be facilitated by the publication. The Advisory Committee hopes that this guide will help local curriculum committees to undertake the construction of their own curriculum guides which will delineate the instructional objectives for each course and each level with a degree of specificity which heretofore has been missing.

The Committee notes with concern the all too frequent pattern found in many schools which reflects the view that the school's curriculum must be equivalent to the textbooks and materials selected for the various offerings. A local curriculum guide should *not* consist merely of a list of topics taken from the tables of content of the adopted texts. Rather, the Committee feels that it is essential for schools to identify appropriate instructional objectives first, and then select the instructional materials, including textbooks, which will contribute to an instructional program designed to achieve those objectives. Each of the instructional objectives should be stated in a behavioral format that spells out the intended behavioral act, the conditions of performance, and the criteria of acceptable performance.

One of the purposes of this publication is to help reverse the practice of selecting textbooks and materials prior to having identified instructional objectives, i.e., intended changes in pupil behavior. The Office of State Superintendent, with the assistance of the Advisory Committee, stands ready to help schools construct their local curriculum guides. *School systems should feel free to duplicate any part of this guide for their own uses.*

The Committee commends this publication to the classroom teachers and administrators of Indiana for their use as they provide quality education for Hoosier boys and girls. It is hoped that teachers and administrators will feel that they have a vital stake in keeping this publication current by providing recommendations for renewal to the Office of State Superintendent.

TEXTBOOK SELECTION

Assisting in textbook selections for his school system is one of the important activities in which a teacher may participate. In Indiana, the books chosen will be used for five years; so many difficulties may be avoided by making careful selections.

The Indiana State Textbook Commission chooses seven books for each course. Then each school corporation makes its selections from this approved list. If a school system desires to offer a course which is not listed or desires to use special books or materials, permission to do so must be obtained from the Office of the State Superintendent of Public Instruction. A letter to the State Superintendent requesting permission should include a course outline, the name of the text desired and the objectives which are desired from this course. This permission must be obtained each year. A follow-up report at the end of the school year is expected by the Office of the State Superintendent.

In order to obtain opinions from several concerned individuals, committees of teachers should make textbook selections. The type of committee organization and the size of the committees will vary with different school systems largely because of the varying size of the systems. These committees should begin work as soon as possible after the Textbook Commission has released the approved list.

On the elementary level, it is often desirable in larger school systems to establish one committee to select the texts for the primary level and another committee for the intermediate grades. The danger in this method is that these committees may select different series which do not provide continuity from the primary to the intermediate levels. The establishment of separate committees for these two levels is worthwhile, but communication should be maintained between them in order to assure concord when the selection is narrowed down to a few possible series.

On the secondary level, it would be desirable in larger school corporations for separate committees to be set up for each area such as General Mathematics, Geometry, Trigonometry, etc. Each of the secondary schools in these larger systems should be represented on each of these subject area committees. The individual school representative should ascertain the thinking of the other mathematics teachers at his school in order to adequately represent his mathematics department.

On the secondary level in smaller school systems it is often difficult to set up special committees for each subject area. In schools where one teacher teaches all sections of the same course, this teacher will probably have the strongest voice in selecting the text for this course. However, great care must be exercised to be certain the selected text is compatible with the choices in other courses. For this reason the complete mathematics department in these schools should meet to discuss the selections.

A sample checklist of basic criteria for evaluation of textbooks is given in this section as a possible aid in selecting textbooks. This list contains four sections: Section I—Philosophy, Section II—Methodology, Section III—Physical Characteristics, and Section IV—Content. The content section will vary from grade to grade and subject to subject and will depend on the course of study. At the elementary level the content objectives can be listed by strands such as *Sets—Numbers—Numerals, Numeration, Operations and Properties, Geometry, Measurement, Graphing and Functions,* and *Probability and Statistics.* At the secondary level the content objectives can be listed by subject. Content objectives can be established from the course of study. Performance objectives established for each of the above strands and for each subject are listed in another section of this publication. These were used in Section IV of the checklist to provide a few sample content items for each of the strands at grade level 3 and also for Algebra I. School systems which do not have local courses of study may duplicate and use the performance objectives given in this guide. The Office of the State Superintendent, with the help of the Advisory Committee, stands ready to help schools construct their local curriculum guides.

In recent years textbook selection committees have often felt it desirable to seek advice concerning the mathematical content after the committee has considered other factors such as appeal to youngsters at the particular level, degree of difficulty for the particular group, teachability, etc. The need for a mathematics consultant is often apparent to elementary school teachers, but secondary teachers may have a similar need. Teachers should not hesitate on any topic to seek such advice.

SAMPLE CRITERIA CHECKLIST FOR EVALUATING TEXTBOOKS

Total Points__________This book ranks__________in a set of__________

Name of Course (grade level)______________________________

Title of book____________________Author(s)________________

Publisher______________________Copyright Date____________

Examiner____________________________Date____________

Directions: Familiarize yourself with the make-up and contents of the text as a whole before beginning the detailed rating. Then rate the book on each item with the following standard in mind. Circle the symbol which identifies your evaluation:

1. Superior	2. Very good	3. Average
4. Barely adequate	5. Unsatisfactory	N/I. Not included but desirable

Delete any items on the checklist which do not apply.

I. Philosophy						
1. Consistent with your course of study	1	2	3	4	5	N/I
2. Presents topics as body of organized knowledge	1	2	3	4	5	N/I
3. Shows relationship to structure	1	2	3	4	5	N/I
4. Consistent use of terminology and symbols	1	2	3	4	5	N/I
II. Methodology						
5. Adequate treatment of all desired topics	1	2	3	4	5	N/I
6. Logical sequence of topics	1	2	3	4	5	N/I
7. Fundamental principles correctly and consistently used throughout	1	2	3	4	5	N/I
8. Develops concepts and skills in a spiral manner	1	2	3	4	5	N/I
9. Clear and accurate definitions	1	2	3	4	5	N/I
10. Explanatory material—clear and precise	1	2	3	4	5	N/I
11. Systematic problem-solving techniques	1	2	3	4	5	N/I
12. Concept of proof adequately presented	1	2	3	4	5	N/I
13. Provision for inductive and deductive reasoning	1	2	3	4	5	N/I
14. Realistic, current verbal problems	1	2	3	4	5	N/I
15. Exercises which are:						
A. Adequate in length and difficulty	1	2	3	4	5	N/I
B. Graded in difficulty (not necessarily in different sections)	1	2	3	4	5	N/I
C. Balance between routine and thought problems	1	2	3	4	5	N/I
D. Provision for independent and creative thinking	1	2	3	4	5	N/I
E. Review, miscellaneous and remedial	1	2	3	4	5	N/I
F. Adequate for oral and mental computation	1	2	3	4	5	N/I

16. A skills development and maintenance program	1	2	3	4	5	N/I
17. Illustrations and examples well chosen	1	2	3	4	5	N/I
18. Examples, illustrations and problems drawn from multi-ethnic contemporary life	1	2	3	4	5	N/I
19. Language level appropriate	1	2	3	4	5	N/I
20. Adaptable to different levels of ability	1	2	3	4	5	N/I
21. Length of chapters and sections appropriate	1	2	3	4	5	N/I
22. Historical notes or reading references	1	2	3	4	5	N/I
23. Career aspects of mathematics adequately treated	1	2	3	4	5	N/I
III. Physical Characteristics and Services						
24. Size and weight	1	2	3	4	5	N/I
25. Cover, binding	1	2	3	4	5	N/I
26. Size, style, clearness of type	1	2	3	4	5	N/I
27. Attractive and functional use of color	1	2	3	4	5	N/I
28. Quality of paper	1	2	3	4	5	N/I
29. Attractive and uncluttered page layout	1	2	3	4	5	N/I
30. Attractive and functional diagrams, drawings, photos, charts	1	2	3	4	5	N/I
31. Relatively free of typographical errors	1	2	3	4	5	N/I
32. Teachers' edition provides:						
A. Adequate format for convenient and effective use	1	2	3	4	5	N/I
B. Clarification and background material	1	2	3	4	5	N/I
C. Aid in methods of presentation, sequencing of topics	1	2	3	4	5	N/I
D. Alternate methods of problem-solving, forms of answers	1	2	3	4	5	N/I
E. Suggestions and materials to aid in diagnosis, remedial work	1	2	3	4	5	N/I
F. Activities, games, projects for enrichment at all levels	1	2	3	4	5	N/I
33. Coordinated test booklets	1	2	3	4	5	N/I
34. Transparencies (masters or film) optionally available	1	2	3	4	5	N/I
35. Solutions manual	1	2	3	4	5	N/I
36. Supplements, remedial or enrichment, in text or separate	1	2	3	4	5	N/I
37. Comprehensive index	1	2	3	4	5	N/I
38. Glossary of terms and/or symbols	1	2	3	4	5	N/I
39. Appropriate tables	1	2	3	4	5	N/I
40. Review and summary materials	1	2	3	4	5	N/I
41. Selected answers optionally available in pupil's edition	1	2	3	4	5	N/I

IV. Content

This section will vary from grade to grade and subject to subject, and will depend on the course of study. Make a checklist of objectives either using those in this guide or those based on a local course of study. Then evaluate the content with respect to these objectives. A few sample items for Grade 3 and for Algebra I follow.

Grade 3

A. Sets—Numbers—Numerals

This textbook provides for learning to:

1. Order a set of numbers (0 to 999) from least to greatest. 1 2 3 4 5 N/I
2. Tell how to distinguish between even and odd cardinal numbers. 1 2 3 4 5 N/I

etc. Appropriate objectives for this strand might be objectives 21 to 32, pages 20-21, in this publication.

B. Numeration

This textbook provides for learning to:

1. Count by 2's, 5's, 10's up to 100 or more. 1 2 3 4 5 N/I
2. Read and write any numeral up to 999. 1 2 3 4 5 N/I

etc. Appropriate objectives for this strand might be objectives 13 to 21, page 26, in this publication.

C. Operations and Properties

This textbook provides for learning to:

1. Recognize a simple multiplicative situation. 1 2 3 4 5 N/I
2. Identify some multiplicative situation. 1 2 3 4 5 N/I

etc. Appropriate objectives for this strand might be objectives 13 to 23, pages 30-31, in this publication.

D. Mathematical Sentences

This textbook provides for learning to:

1. Write subtraction sentences which result from a given addition statement and vice versa. 1 2 3 4 5 N/I
2. Determine whether a mathematical sentence is true or false. 1 2 3 4 5 N/I

etc. Appropriate objectives for this strand might be objectives 8 to 15, pages 38-39, in this publication.

E. Geometry

This textbook provides for learning to:

1. Distinguish between geometric abstractions and models of them.	1	2	3	4	5	N/I
2. Identify the circle, triangle, square, rectangle, and other simple polygons, recognizing some of their properties.	1	2	3	4	5	N/I

etc. Appropriate objectives for this strand might be objectives 7 to 18, pages 41-42, in this publication.

F. Measurement

This textbook provides for learning to:

1. Compare the values of U.S. coins to 50¢.	1	2	3	4	5	N/I
2. Use and compare common length measures such as inch, foot, and yard.	1	2	3	4	5	N/I

etc. Appropriate objectives for this strand might be objectives 15 to 26, pages 48-49, in this publication.

G. Graphing and Functions

This textbook provides for learning to:

1. List a rate equivalent to a given rate.	1	2	3	4	5	N/I
2. Use the number line to illustrate a multiplication or division fact.	1	2	3	4	5	N/I

etc. Appropriate objectives for this strand might be objectives 3 to 6, page 51, in this publication.

H. Probability and Statistics

This textbook provides for learning to:

1. Use average as a number that helps describe a group.	1	2	3	4	5	N/I
2. Interpret the information in a bar graph.	1	2	3	4	5	N/I

etc. Appropriate objectives for this strand might be objectives 4 to 9, pages 55-56, in this publication.

Algebra I

This textbook provides for learning to:

1. Map the set of real numbers onto the number line.	1	2	3	4	5	N/I
2. Identify and state the order relations on the set of real numbers.	1	2	3	4	5	N/I

etc. The full list of objectives for Algebra I can be found on pages 69-70 of this publication.

Total I	Philosophy	________
Total II	Methodology	________
Total III	Physical Characteristics and Services	________
Total IV	Content (these totals vary from course to course and grade to grade as the number of items will vary)	________
	GRAND TOTAL	________ (enter on front)

Low totals are desirable. Rank evaluations of all books in the same subject in order of totals.

Check here if this book has particularly undesirable characteristics.________

Explanation:____

____

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____

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Note below any particularly outstanding characteristics.________________

____

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Considering all the preceding features of this textbook (series) my over-all recommendation is____

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